Problem Set A
Economics of Digital Markets

Note: You can earn up to 25 points. Points for a problem are only awarded for (almost) completely correct solutions to a problem.

Problem 1 (Personalized Pricing, 10 points)

Consider a monopolist that sells a single good. The demand for the good is represented by the inverse demand function (price, p, as function of quantity, q) $p = 10 - 2q$. Suppose that there are no costs of production.

a) Write down the profit function of the monopolist. Suppose that the monopolist maximizes profits by choosing its optimal quantity. Write down the maximization problem of the monopolist, the associated First Order Condition (FOC), and check the Second Order Condition (SOC). Then solve for the optimal quantity of the monopolist, verify that this is $q^m = 2.5$.

b) Calculate the price that results at q^m. Represent this market in a graph with quantity on the $x-axis$ and price on the $y-axis$: show the inverse demand function and indicate the areas in the graph that represents the consumer surplus (CS) and the producer surplus (PS) at the monopoly outcome.

c) Calculate consumer surplus, producer surplus, and total surplus for the monopoly outcome. Do the same, i.e., calculate CS, PS, and total welfare, for the outcome under 'perfect competition', when price equals marginal costs. Compare the two modes of competition, monopoly and perfect competition, by calculating the Dead Weight Loss, the loss in total surplus under monopoly compared to 'perfect competition'.

d) Suppose that demand consists of individual consumers whose willingness to pay is given by the value of the inverse demand function. Suppose the monoppolist can identify whether a consumer has a willingness to pay below or above 3. The monopolist can set two different prices to the two resulting groups. Determine the demand for each of the two groups. Then calculate the optimal price for each group for the monopolist.

e) How does this information in part d) affect CS and PS? How are the two consumer groups affected?

Problem 2 (Tracking, 5 points)

A monopolist sells a single good in two periods. There are two conumers who want to buy one unit of the good in each period. The willingnessto pay of consumer A is 2, while the willingness to pay of consumer B is 1.5.

a) Suppose the monopolist can only set one price in each period and cannot identify consumers at all. What is the optimal price?

b) Suppose that the monoplist has indetified which consumers is A and B in period two and can set different prices. What are the optimal prices?

c) Would the monoplist want to set a price of 2 in the first period, if he can identify the consumer who bought the product in the next period. That is if he can track who has bought in the first period. Assume for the moment that consumer A buys in the first period at a price of 2.

d) Assume that the willingness to pay of consumer B is 0.5. Suppose the monopolist can only set one price in each period and cannot identify consumers at all. What is the optimal price? Would consumer B profit from providing information to the monopolist that verifies his willingness to pay? Relate your answer to the concept of *information unravelling*.

Problem 3 (Targeted advertising, 10 points)

Consider a market with two firms. Each firm is located at one end of a line with lenght one. There is a mass one of consumers. The location of each consumer is given by $0 < x < 1$ which is uniformly distributed (with density 1). Firms have no cost of production and set price simultaneously.

a) Derive the demand for each firm by identifying the location of the indifferent consumer for each price pair. Assume that all consumers know about both products.

b) Write down the profit functions and calculate the Nash equilibrium prices for both firms.

c) Assume that consumers only know the product if they have received and ad. Suppose that ads are not targeted and each firm reaches any consumer with probability 0.5 with her ad. Calculate the size of the different consumer segments. Determine the resulting demand and the new Nash equilibirum prices of the firms.

d) Suppose that the ads are costless. When do the firms make larger profits? With fully informed consuemers b) or with imperfect ads c)?

e) Consider again that consumers can only buy after receiving an ad. Suppose there is an avdertising company that offers the firms to coordinate the targeting of their ads. The company suggests to inform all consumers with a location between 0 and 0.4 the product of the firm at location 0 and to all consumers between 0.6 and 1 the product of the firm at location 1. Determine the optimal prices for both firms if they accept this offer. What are the resulting profits?