Econ 104: Intro to Econometrics
SPRING 2020

Problem Set 4a (Due: March 30, 2020)

Problem 1

Suppose that the data generating process is determined by the following Cobb-Douglas production function

$$\log Y_i = \alpha_0 + \alpha_1 \log L_{1i} + \alpha_2 \log L_{2i} + \alpha_3 \log K_i + u_i$$

where Y_i = output of firm i, L_{1i} = production labor of firm i, L_{2i} = non-production labor of firm i, and K_i = capital stock of firm i. Suppose you instead use OLS to estimate the following model:

$$\log Y_i = \beta_0 + \beta_1 \log L_{1i} + \beta_2 \log K_i + u_i$$

using cross-section data on many firms.

1. Will $E[\hat{\beta}_1] = \alpha_1$ and $E[\hat{\beta}_2] = \alpha_3$? Explain.
2. You think that the error term may be heteroskedastic so you use heteroskedasticity-robust standard errors for $\hat{\beta}_0$, $\hat{\beta}_1$, and $\hat{\beta}_2$. Would they be valid for testing hypotheses about true values of α_1 and α_3? Explain.
3. How will your answers to parts 1 and 2 change if it is known that $\alpha_2 = 0$, i.e. L_2 is an irrelevant input in the production function? Explain.
4. Assuming $\alpha_2 = 0$, explain how you would test for constant returns to scale.
   Hint: See SW Problem 7.9c

Problem 2

Suppose that the data generating process is now determined by the following Cobb-Douglas production function:

$$\log Y_i = \beta_0 + \beta_1 \log L_{1i} + \beta_2 \log K_i + u_i \tag{1}$$

where Y_i = output of firm i, L_{1i} = production labor of firm i, and K_i = capital stock of firm i. Suppose you instead use OLS to estimate the following model:

$$\log Y_i = \alpha_0 + \alpha_1 \log L_{1i} + \alpha_2 \log L_{2i} + \alpha_3 \log K_i + u_i \tag{2}$$

where L_{2i} = non-production labor of firm i using cross-section data on many firms.

1. Will $E[\hat{\alpha}_1] = \beta_1$ and $E[\hat{\alpha}_3] = \beta_2$? Explain.

2. You think that the error term may be heterokedastic so you use heteroskedasticity-robust standard errors for $\hat{\alpha}_0$, $\hat{\alpha}_1$, $\hat{\alpha}_2$, and $\hat{\alpha}_3$. Will the standard errors be valid for testing hypotheses about true values of β_0, β_1 and β_2? Explain.

3. Suppose you estimate both models (1) and (2) by OLS using cross-sectional data on **very** many firms. You test $H_0 : \alpha_1 = 1$ against $H_1 : \alpha_1 \neq 1$ using the t-statistics

$$t = \frac{\hat{\alpha}_1 - 1}{s.e.(\hat{\alpha}_1)}$$

and $H_0 : \beta_1 = 1$ against $H_1 : \beta_1 \neq 1$ using

$$t = \frac{\hat{\beta}_1 - 1}{s.e.(\hat{\beta}_1)}$$

and the decision rule "reject H_0 if $|t| > 1.96$" (since 1.96 is the 5% critical value). Suppose H_0 is true. Are you more likely, less likely, or equally likely to reject H_0 using the first t statistic (involving $\hat{\alpha}_1$) than using the second t statistic (involving $\hat{\beta}_1$)? Explain.

4. Consider the setup from part 3, but suppose that H_0 is false. Are you more likely, less likely, or equally likely to reject H_0 using the first t statistic (involving $\hat{\alpha}_1$) than using the second t statistic (involving $\hat{\beta}_1$)? Explain.