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Chapter 9.2, Problem 20Q

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Problem

Two popular forms of mortgage are the thirty-year fixed-rate mortgage, where the borrower has thirty years to repay the loan at a constant rate, and the adjustable rate mortgage (ARM), one version of which is for five years with the possibility of yearly changes in the interest rate. Since the ARM offers less certainty, its rates are usually lower than those of fixed-rate mortgages. Test this hypothesis at the $\alpha = 0.01$ level using the following sample of mortgage offerings for a loan of \$250,000. Do not assume the variances are equal.

\$250,000 Mortgage Rates

30-Year Fixed	ARM
3.525	2.923
3.625	3.385
3.383	3.154
3.625	3.363
3.661	3.226
3.791	3.283
3.941	3.427
3.781	3.437
3.660	3.746
3.733	3.438

Step-by-step solution

Step 1 of 7

The data table as given in the question:

30-year Fixed	ARM
3.525	2.923
3.625	3.385
3.383	3.154
3.625	3.363
3.661	3.226
3.791	3.283
3.941	3.427
3.781	3.437
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For the 30-year fixed mortgage:

The sample size $n_1 = 10$

The sample mean is:

$$\begin{aligned}\bar{x}_1 &= \frac{3.525 + 3.625 + \dots + 3.733}{10} \\ &= \frac{36.725}{10} \\ &= \boxed{3.6725}\end{aligned}$$

The sample standard deviation is:

$$\begin{aligned}s &= \sqrt{\frac{(3.525 - 3.6725)^2 + (3.625 - 3.6725)^2 + \dots + (3.733 - 3.6725)^2}{10 - 1}} \\ &= \sqrt{\frac{(-0.1475)^2 + (-0.0475)^2 + \dots + (0.0605)^2}{9}} \\ &= \sqrt{\frac{(0.021756 + 0.002256 + \dots + 0.00366)}{9}} \\ &= \sqrt{\frac{0.211935}{9}} \\ &= \sqrt{0.023548} \\ &= \boxed{0.1535}\end{aligned}$$

For the ARM mortgage:

The sample size $n_2 = 10$

The sample mean is:

$$\begin{aligned}\bar{x}_2 &= \frac{2.923 + 3.385 + \dots + 3.438}{10} \\ &= \frac{33.382}{10} \\ &= 3.3382\end{aligned}$$

The sample standard deviation is:

$$\begin{aligned}s_2 &= \sqrt{\frac{(2.923 - 3.3382)^2 + (3.385 - 3.3382)^2 + \dots + (3.438 - 3.3382)^2}{10 - 1}} \\ &= \sqrt{\frac{(-0.4152)^2 + (0.0468)^2 + \dots + (0.0998)^2}{9}} \\ &= \sqrt{\frac{(0.17239 + 0.0021 + \dots + 0.0099)}{9}} \\ &= \sqrt{\frac{0.4186}{9}} \\ &= \sqrt{0.04651} \\ &= \boxed{0.2157}\end{aligned}$$

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Step 3 of 7

The null and the alternative hypothesis is given by:

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 > 0 \quad (\text{Right-tailed test})$$

Here, μ_1, μ_2 represent the true mean rates of 30-year fixed mortgage and ARM mortgage respectively.

The level of significance $\alpha = 0.01$



Step 4 of 7

It is given in the question to not assume the variances are equal. Therefore, use the t -test for unequal population variances. Which is given by:

$$\begin{aligned}
 t &= \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \\
 &= \frac{3.6725 - 3.3382}{\sqrt{\frac{(0.1535)^2}{10} + \frac{(0.2157)^2}{10}}} \\
 &= \frac{0.3343}{\sqrt{0.002355 + 0.004652}} \\
 &= \frac{0.3343}{\sqrt{0.007007}}
 \end{aligned}$$

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Step 5 of 7

$$\begin{aligned}
 &= \frac{0.3343}{0.083706} \\
 &= \boxed{3.99}
 \end{aligned}$$

The degrees of freedom for unequal variance t -test is given by:

$$\begin{aligned}
 df &= \frac{\left[\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right]^2}{\frac{\left(\frac{s_1^2}{n_1} \right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{n_2 - 1}} \\
 &= \frac{\left[\left(\frac{(0.1535)^2}{10} \right) + \left(\frac{(0.2157)^2}{10} \right) \right]^2}{\frac{\left(\frac{(0.1535)^2}{10} \right)^2}{10 - 1} + \frac{\left(\frac{(0.2157)^2}{10} \right)^2}{10 - 1}} \\
 &= \frac{(0.002355 + 0.004652)^2}{0.000000616 + 0.0000024} \\
 &= \frac{0.0000491}{0.00000302} \\
 &= 16.25 \\
 &\sim \boxed{16} \text{ (Rounded to the nearest integer.)}
 \end{aligned}$$

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Step 6 of 7

At $t = 3.99$ and 16 degrees of freedom the p -value of the hypothesis test is $\boxed{0.001}$

(By using the t -table for a one-tailed test.)

Here, the p -value $<$ the level of significance

$$0.001 < 0.01$$

Therefore, 'reject the null hypothesis'.

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Conclusion:

At 0.01 level of significance, there is sufficient evidence to conclude that the ARM mortgage offers less certainty as compared to the 30-year fixed mortgage.

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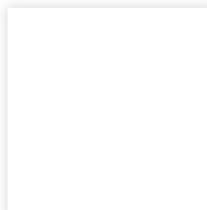
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