

Consider a market where you can trade 3 shares and cash. The the price of the shares at the beginning are 100 and their price is based on the following formula:

$$S_{t+1}^i = 0.5 \cdot 100 + 0.5 \cdot S_t^i + \xi^i - \exp(2.5)$$

where ξ is a lognormal distribution with parameters $(2; 1)$

Consider the following derivative: The issuer agrees to exchange one Share 1 to one Share 2 at their maturity date if the contracting party so requests. First let's look at the process for only one period. Let there be 100 branches. (decision trees). What is the price of the derivative? Could the derivative be cheaper?

Let there be 1000, 10000, 100000 branches. What is the price of the derivative? Let's look at the process for 2 periods. Let there be 100x100 branches. What is the price of the derivative now?

Let's look at the process for 3 periods. Let here be n branches in every time period. How does the price of the derivative change? For what n can we solve the problem?