ECON3209: Statistics for Econometrics
Assignment 2

INSTRUCTIONS

Due date and time: FRIDAY 23 rd April by 11:55PM

Total weight in final assessment: 15%

Total marks for the assessment: 40. Each question part has equal mark value.

Marking: *Marks will be awarded for correct working and explanation as well as correct final answers.* Students are allowed to work in groups. However, each student MUST hand in their own work in their own words. Plagiarism is a university offence and will be checked.

Submission: Students must submit 1 electronic copy of their assignment. The electronic copy is to be submitted to the course website via 'Turnitin' by the due date and time. Upload a copy of your document as a PDF – **do not paste text.**

Assignment: The assignment must be produced using an appropriate word-processor (no exceptions). The assignment must have a completed and signed official cover sheet as the first page of the document. This can be found on the course website in the 'course documents' folder. Each question (Question 1 and Question 2) should be started on a new page. At the end of each question, you must provide a copy of the Stata programs requested.

CONTEXT & DATA

The Republic of Statistonia has many weather stations, which have recorded annual rainfall over many years. The assignment questions below ask you to analyze rainfall data for one weather station, to draw inferences and discuss results. To undertake the analysis you will be required to use Stata.

The data set for all weather stations is contained in a Stata data file named Rainfall_v1.dta, which is located in the Assignment folder on Moodle. Click on it to save in your own working directory. If you move to this directory in Stata (using the 'cd directory name' command), then you can upload the data file into Stata by the 'use Rainfall_v1.dta' command. The data set comprises 50 observations on annual rainfall (in metres) at each of the weather stations. The first variable in the data set is the year and the remaining variables for rainfall by weather station are labeled Rainfall1, Rainfall2, and so forth. Your weather station data are identified by examining the file Stations.pdf, also located in the Assignment folder. It lists student zIDs and the corresponding rainfall variable that you are to use.

Questions
[Each question part has equal mark value]

1. Suppose that random variable $X \geq 0$, representing the annual rainfall (in metres) at your weather station, has a Gamma distribution, i.e., $X \backsim Gamma(\alpha, \beta)$, where α and β are positive parameters. Below, we re-parameterize by using the parameter $\theta = 1/\beta$.

   (a) Use Stata to describe your rainfall data series, by using the 'summarize' command and plotting the kernel density function. Explain and discuss these results, including the nature and implications of the empirical distribution for rainfall at your weather station.

   (b) Estimate the parameters of the Gamma distribution by the method of maximum likelihood, presenting your results (parameter estimates, standard errors, etc.) in tabular form. [Hint: You may wish to use the 'mlexp' command. The syntax for the command is of the form: mlexp (ln(gammaden({alpha},{theta},0,xvar))), where 'xvar' is the variable name.] [Note: In Stata, the gamma density function 'gammaden' uses α and $\theta = 1/\beta$ as the parameters, along with a location parameter set to 0.]

   (c) Plot the estimated (using the maximum likelihood estimates) Gamma density for rainfall. Using the estimated Gamma density for rainfall, compute estimates of the probabilities of rainfall being (i) less than 70% of average annual rainfall (very dry), (ii) more than 130% of average annual rainfall (very wet) and (iii) either (i) or (ii). Discuss these results and their implications.

   (d) Use the likelihood ratio test procedure to test the null hypothesis that $\theta = 1$ (meaning that the population mean and variance are equal) against the alternative hypothesis that $\theta \neq 1$ using a 5% significance level. Your answer should provide full details of the logic used and your conclusion. [Hint: To estimate the model with $\theta = 1$, replace '{theta}' by '1' in the 'mlexp' command.]

   Include your Stata program(s) at the end of your answers to this question.

2. Again, suppose that random variable $X \geq 0$, representing the annual rainfall (in metres) at your weather station, has a Gamma distribution, i.e., $X \backsim Gamma(\alpha, \beta)$, where α and β are positive parameters.

   (a) Obtain the method of moments estimates for the two parameters. Using these estimates, calculate estimates of the population mean and standard deviation.

   (b) Construct and undertake a Bootstrap simulation of the model to determine the small sample ($n = 50$) properties of the method of moments estimators of α and β for your rainfall data. Provide a summary table for the simulation results, describing and explaining its contents.

   (c) Present a table of your results for the method of moments parameter estimates based upon your rainfall data and their bootstrap standard errors. Explain how you obtained the bootstrap standard errors.

   (d) Plot the kernel density function, along with the normal density, for each of the parameter estimates obtained from the simulation. What do these results tell you about the distribution for the method of moments estimators for annual rainfall and the implications for hypothesis testing?

   Include your Stata program(s) at the end of your answers to this question.
