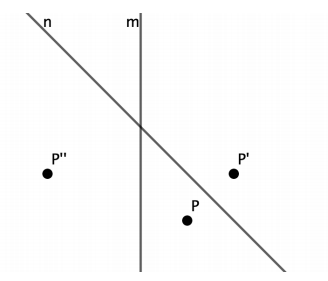
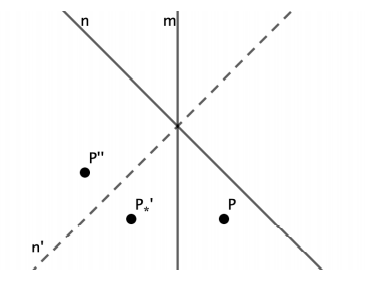
**Inverted mirrors**



Point P is reflected in line n and then in line m. The result is point p ′′:



There is another way to arrive at point p ′′: first mirror both point P and line n in line m; that gives point p"∗ and line n′. Continuing mirror p"∗ in n ′



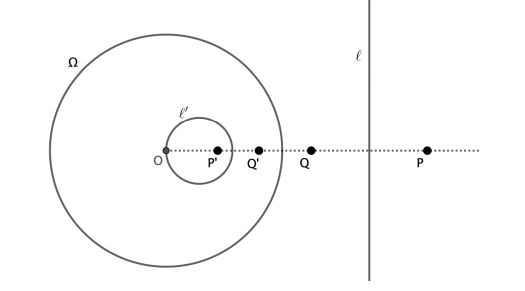
You can imagine standing in front of a large mirror wall with a hand mirror ...

Does a similar property also apply to inversion? That is what this assignment is about. We have considering that inversion is not really mirroring in a circle

(that is, if you really have one circular mirror, he does a lot differently with the world than inversion does). But in In some ways inverting is very similar to mirroring in a circle!

**Exercise:**

Given are a circle Ω with center O, a point P and a line ℓ such that ℓ is perpendicular to OP. If we mirror P in ℓ we find point Q. (See drawing).



We invert the whole picture in the circle Ω. We call the image under inversion of P We call p′, the image of ℓ, with point O added to it, ℓ ′ (note: ℓ ′ is a circle) and the image of Q we call Q ′.

Proof: If we “mirror” point p ′, that is, invert, in circle ℓ ′, we find point Q ′.

Mirroring in ℓ changes under inversion to invert in ℓ ′

**Question:**

What happens in the preparation situation if we replace the line ℓ with an arbitrary circle Γ and replace mirror in ℓ with invert in Γ?

**Suggestions:**

• Formulate your answer in the form of a suspicion and first try to convince yourself that the suspicion is correct. Then try to really prove your suspicion.

• Investigate special cases. For example, first the case from the preparation above, and then a situation where line ℓ is not perpendicular to OP.

• You have different types of tools: analytical for example (calculating with distances and coordinates). But you also know what inversion does to lines and circles; and that intersection points of two objects are also after inversion intersection of the two inverted objects ...

• It is a problem approach assignment. Of course, a complete solution is appreciated, but you can also get a good score if you clear your approach describes, shows variation and looks back on it carefully. For this, study the assessment rubric (see below).

**This mathematical problem must be tackled by using the four phases of Pólya. You describe your entire process, which means that - which is unusual in mathematics education - you also describe failed attempts and analyze why no success was achieved here. You can give your text the chapter division according to the phases of Pólya, but because in practice you often jump back and forth between phases, you can also report on it as long as you indicate which phase you are in. You also add a reflection on what you have learned as a problem solver at the end. Here you try to take some distance from the specific problem and, for example, give yourself tips for what to do next.**