

Problem Set 5 (Due on 04/13, in class)

This problem set reviews wavelets and time-frequency analysis

Total points: 50pt

Problem 1. JPEG-2000 (10pt)

Many of you have heard about the JPEG format of images. Not so many have heard about the JPEG-2000 format.

Read https://en.wikipedia.org/wiki/JPEG_2000 and answer the following questions.

1. What is the main difference between JPEG-2000 and JPEG?
2. Why is JPEG-2000 never commonly adopted as JPEG?

Problem 2. Programming: wavelet decomposition (10pt)

Take your favorite picture and preprocess it to a grayscale image. Then perform a single-level 2-D wavelet decomposition using your favorite wavelet package (e.g. `pywavelets` in python). You need to show four images which correspond to the approximation, the horizontal detail, the vertical detail and the diagonal detail, respectively. You can choose to use any wavelet (Haar, Shannon, Daubechies, etc.).

Please submit both your codes and results.

Problem 3. Programming: Dow Jones (10pt)

In this task, you are going to study the Dow Jones Industrial Average Index data over 25 years.

To retrieval the data, go to <https://finance.yahoo.com/quote/%5EDJI/history/>, choose "MAX" for Time Period and "Daily" for Frequency. You can download the data and retrieve a time series of the [daily close prices](#).

Plot the spectrogram of the time series of the daily close prices. For instance, you can use "`scipy.signal.spectrogram`" in python or "`spectrogram`" in MATLAB. You can choose to use your favorite window function.

Please submit both your codes and results.

Problem 4. Shannon MRA (10pt)

Verify that the Shannon approximation gives an MRA by showing Properties 1, 2, 3 in the definition of the MRA.

Problem 5. Orthonormal basis (10pt + (bonus) 5pt)

1. Prove: in an MRA, the family of $\{\psi_{j,n}\}_{j,n \in \mathbb{Z}^2}$ is an orthonormal basis of $L^2(\mathbb{R})$.
2. (Bonus) Given an example of a function $\psi \in L^2(\mathbb{R})$ such that

$$\sum_{j=-\infty}^{\infty} |\hat{\psi}(2^j \omega)|^2 = 1, \quad \text{for all } \omega \neq 0,$$

but $\{\psi_{j,n}\}_{j,n \in \mathbb{Z}^2}$ is not an orthonormal basis of $L^2(\mathbb{R})$.