## PROBLEM # 1

The Hagen-Poiseuille law gives the flow rate equation for a horizontal steady-state flow of an incompressible liquid in a circular tube of radius R, and length L, subjected to a pressure drop $\Delta$P.

1. Use the Hagen-Poiseuille equation as a starting point to derive an expression for a porous medium permeability (k) that has a porosity $\phi$, tortuosity $\tau$, and a pore radius distribution $f_p$. Assume the porous medium consists of a cylindrical core sample of length L, and cross sectional area A.

Assume that the capillary tube model is adequate for simulating fluid flow in porous media, and assume that the capillary-tube radii distribution is also $f_p$ which is a function of tube radii. Assume the fluid flow is horizontal.

Hagen-Poiseuille is given by
$$q = \frac{\pi r^4 \Delta P}{8 \mu l}$$

l      is the length of a given capillary

$\mu$      is the viscosity of the fluid flowing in the porous medium

r      a tube radius

P      stands for pressure

2. Using results of part 1, estimate the porous medium permeability in md for a porosity value of 35%, tortuosity value of 2, and a pore radius distribution $f_p$ given by

$$f_p = \frac{1}{r} + \frac{0.5}{r^2} \qquad\qquad 10^{-6} m \le r \le 10^{-5} m$$

Use    $10^{-12}$ m$^2$ = 1Darcy.

**PROBLEM # 2**

Using the flow rate equation through a rectangular slit, given by

$$Q = \frac{2}{3}\left(\frac{\Phi_0 - \Phi_L}{\mu L}\right) B^3 W.$$

Show that the permeability through a fractured rock simulated by a bundle of slits is given by

$$k = \frac{\phi}{3\tau^2} \frac{\displaystyle\int_0^B B^3 f_p\, dB}{\displaystyle\int_0^{\dot{B}} B\ f_p\, dB}$$
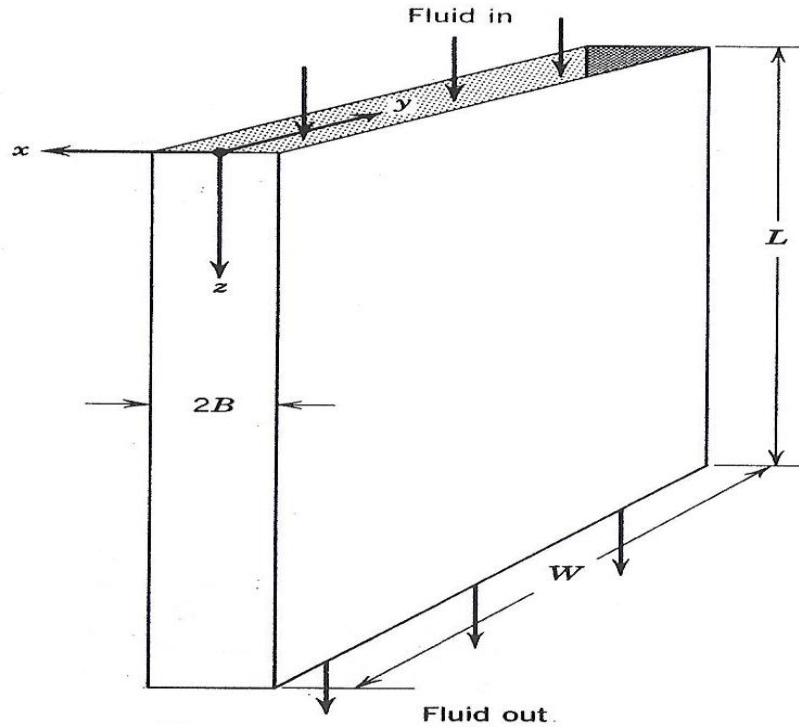
Use the geometry specified Figure 1 below.



Figure 1. Flow through a slit.