Q1:

Suppose that the following constraints have been provided for a linear programming model with decision variables x_1 and x_2.

$$-x_1 + 3x_2 \leq 30$$

$$-3x_1 + x_2 \leq 30$$

and

$$x_1 \geq 0 \qquad x_2 \geq 0$$

i) Demonstrate graphically that the feasible region is unbounded.

ii) If the objective is to maximize $Z = -x_1 + x_2$, does the model have an optimal solution? If so, find it. If not, explain why not.

iii) Repeat part (ii) when the objective is to maximize $Z = x_1 - x_2$.

iv) For objective functions where this model has no optimal solution, does this mean that there are no good solutions according to the model? Explain. What probably went wrong when formulating the model?

v) Select an objective function for which this model has no optimal solution. Then work through the simplex method step by step to demonstrate that Z is unbounded.

Assignment

Q1

[redacted] **HYDROELECTRIC POWER SYSTEMS PLANNING**

An agency controls the operation of a system consisting of two water reservoirs with one hydroelectric power generation plant attached to each as shown in Fig. 2.1. The planning horizon for the system is broken into two periods. When the reservoir is at full capacity, additional inflowing water is spilled over a spillway. In addition, water can also be released through a spillway as desired for flood protection purposes. Spilled water does not produce any electricity.

Assume that on an average 1 kilo-acre-foot (KAF) of water is converted to 400 megawatt hours (MWh) of electricity by power plant A and 200 MWh by power plant B. The capacities of power plants A and B are 60,000 and 35,000 MWh per period. During each period, up to 50,000 MWh of electricity can be sold at \$20.00/MWh, and excess power above 50,000 MWh can only be sold for \$14.00/MWh. The following table gives additional data on the reservoir operation and inflow in kilo-acre-feet:

	Reservoir A	Reservoir B
Capacity	2000	1500
Predicted inflow		
Period 1	200	40
Period 2	130	15
Minimum allowable level	1200	800
Level at the beginning of period 1	1900	850

Develop a linear programming model for determining the optimal operating policy that will maximize the total revenue from electricity sales.

Q3:

Consider the following problem:

Minimize $Z = 2x_1 + 3x_2 + x_3$,

Subject to

$$x_1 + 4x_2 + 2x_3 \geq 8$$

$$3x_1 + 2x_2 \geq 6$$

and

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0.$$

i) Reformulate this problem to fit the canonical form for a linear programming model.
ii) Using the Big M method, work through the simplex method step by step to solve the problem.
iii) Using the two-phase method, work through the simplex method step by step to solve the problem.
iv) Compare the sequence of BF solutions obtained in parts (b) and (c). What conclusion you can derive from this comparison?