## 14

Suppose that population is distributed according to

$$n(x) = \mathbb{1}_{\{\text{mod }(x_1,\ 2)=0\}}(x) + \mathbb{1}_{\{\text{mod }(x_1,\ 4)=0\}}(x),$$

where $x \in X := \{0, \pm 1, \pm 2 \cdots, \pm m\}^2$ and $\mathbb{1}_S(\cdot)$ is an indicator function. Suppose that the census is tabulated according to two types of tracts:

1. $C_i = \{x \in X : x_1 = i\}$, and
2. $R_j = \{x \in X : x_2 = j\}$,

where $i,\ j \in \{0, \pm 1, \pm 2 \cdots, \pm m\}$.

1. Suppose that a city is defined to be the tract where population density is greater than 2. Map out the city locations according to each type of tracts.
2. Repeat item 1 assuming that the threshold population density is 4 rather than 2.
3. Are cities in this example unit sensitive? Explain.

## Set VII

### 15

1. Propose a fractal whose dimension is $\frac{\log 5}{\log 3}$.
2. Propose a fractal whose dimension is $\frac{\log 6}{\log 4}$.
3. Between the two fractals above, which one fills more spaces?

### 16

Consider a polycentric city with the population distributed according to this data set. The first row marks the longitude and the first column marks the latitude. 1 indicates a resident and 0 indicates vacancy. Demarcate the area into tracts[1] to compute the population density $D(m)$, and map out its level set. Which of the three polycentric formations introduced in class best describes the density in this city?

### 17

Consider a polycentric city with the population distributed according to this data set. It is tabulated in the same way as section 16. There are two subcenters located at $m^1 = (-60, 0)$ and $m^2 = (60, 0)$, where the first entry is the longitude and the second is the latitude. Compute the population density $D(m)$ and regress it on the distances from each subcenter assuming the second form (complements) in class and estimate the population gradients $a^1$ and $a^2$.

## Set VIII

### 18

Find Nash equilibia (if any) for $N = 3$ in Hotelling's model.

### 19

Consider a Hotelling's model with two vendors introduced in class with a unit circle instead of a unit line segment. Consumers are evenly distributed along the circumference. Distance is measured by item 2 in section 1. Two vendors can pick a location along the circumference. Find the Nash equilibrium.

### 20

Consider a closed linear monocentric city stretching from $r = 0$ to $N(> 0)$ with land supply $L(r) = 1$ for any $r$. In line with the theme of this segment (**inter**urban economics), interpret the segment $[0, N]$ as Canada itself rather than one particular city, with $r = 0$ being the west coast and $r = N$ the east coast for example.

Suppose that consumers are exogenously distributed according to either

$$n_U(r) = 1, \quad \text{or}$$

$$n_D(r) = \begin{cases} N & \text{if } r = 0 \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

1. Find land consumption $s_U(r)$ and $s_D(r)$ under each distribution.

Endowments are uniformly distributed across the country (rather than only at $r = 0$ as in the usual monocentric city model), namely, $w(r) = w$ anywhere in the country. A consumer **cannot** relocate but she picks up her endowment $w$ following the same commuting cost schedule as section 4. Consumers have homogeneous preferences represented by

$$u(s(r),\ z(r)) = \log(s(r)) + z(r), \tag{2}$$

2. Denote the bid-rent function by $p_U(r)$ or $p_D(r)$ respectively. Find the equilibrium for each one of (1).
3. Between the equilibrium with $n_U(r)$ and $n_D(r)$, which one Pareto improves upon the other?

### 21

Consider the New Economic Geography model introduced in class.

1. Show that the price index is given by

$$G = \left( \int p(i)^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

2. Show that the profit-maximizing price is

$$p(i) = \frac{\sigma}{\sigma - 1} w(i) c.$$

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[1] Of your choice. 10-by-10 square tracts are probably easy to work with.

## Set IX

### 22

Suppose that manufacturing workers inexplicably follow the dynamics below:

$$\omega_1 - \omega_2 > 0 \quad \Rightarrow \quad \lambda \text{ decreases}$$
$$\omega_1 - \omega_2 = 0 \quad \Rightarrow \quad \lambda \text{ stays the same}$$
$$\omega_1 - \omega_2 < 0 \quad \Rightarrow \quad \lambda \text{ increases}$$

Using the graphs in Graph Vault, examine if the economy still yields pitchfork bifurcation.

### 23

Run rank-size regression on US Metropolitan Statistical Area in 2010 (data), Places, and Urban Areas (data). Is the rank-size rule scale sensitive?

## Set X

Due April 17th.

### 24

Solve section 16.

### 25

Solve section 17.

### 26

Consider the Eeckhout's model of a city-size distribution with two cities. Consumer preferences are represented by

$$u\left(c_t^i,\ h_t^i,\ l_t^i\right) = \left(c_t^i\right)^{\alpha} \left(h_t^i\right)^{\beta} (1 - l_t^i)^{\gamma},$$

where $c_t^i$ is a numéraire composite goods consumption and $h_t^i$ is housing consumption. Each consumer is endowed with a unit of time, out of which they spend $l_t^i$ on work and the remainder on leisure. Normalize $\alpha + \beta + \gamma = 1$. Housing price is $p_t^i$ per unit and each city has $H$ units of land for residential use. Wage rate is $w_t^i$ per unit of effective labor defined by $L_t^i := \left(S_t^i\right)^b l_t^i$, where $b < 0$ measures the degree of negative externalities (i.e., $a_-\left(S_t^i\right) = \left(S_t^i\right)^b$).

Suppose that city 1 drew a better technology in $t = 0$ so that $A_0^2 = \dfrac{A_0^1}{2}$. Is $S_0^2 = \dfrac{S_0^1}{2}$ as well? Solve for the equilibrium and explain.