ASSIGNMENT 4

CEE/ENGMT 5980 Decision Analysis

## Instructions

In this assignment, you will:

* Examine the implications of using an a priori method
* Utilize an advanced visualization tool to explore tradeoffs in the GAA dataset
* Bonus: conduct an OAT and global sensitivity analysis

Please turn in a separate pdf document containing the answers to each of the parts of this assignment

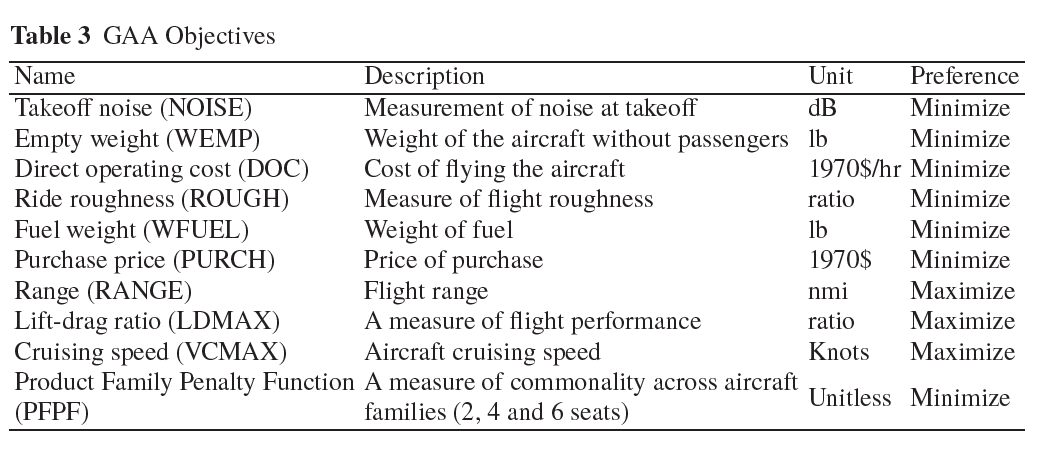
**Note:** Though your work will only be seen by those grading the course and will not be used or shared outside the course, you should take care to obscure any information you feel might be of a sensitive or confidential nature.

You can complete each assignment exercise as you progress through the unit. Wait to submit the assignment until all exercises are complete. A **Submit Assignment** button can be found on the assignment page. Information about the grading rubric is available on any of the assignment pages online. Do not hesitate to contact your TA if you have any questions about the assignment.

Part One

# Examining solutions from the GAA

You are a consultant working with the American Aviation Company (AAC) to design a general aviation aircraft (GAA). You’ve run a multi-objective optimization using the 10 objective formulation described by Woodruff et al., (2013). A description of each of the 10 objectives can be found in the table below. To make things simpler, all objectives that are being maximized have been made negative, so that universally lower values are preferred for all objectives. (Note: It’s recommended to use Excel, Python or Matlab for parts a-c)



1. Using the goal programming methodology described in Woodruff et al., (2013) equation 2, select a solution from the Pareto set using the following goals. Ignore objectives that do not have goals and you do not need to worry about separating the 2, 4 and 6 seat metrics.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Objective Name:** | NOISE | WEMP | DOC | ROUGH | WFUEL | PURCH | RANGE | LDMAX | VCMAX | PFPF |
| **Goal:** | NA | 1950 | 60 | NA | 400 | 42000 | -2500 | -17 | -200 | NA |

What ID number is selected? Describe this solution. (Note: for this part of the problem, use the file named “GAA\_pset\_IDs.csv”). (5 points)

1. Plot the goal programming performance against the PFPF objective and note the location of the solution selected in part (a). Highlight the Pareto Front, is there a tradeoff between the two performance measures (5 points)?
2. Does the goal programming solution lie on the Pareto front? What does its location say about the a priori method (2 points)?
3. Now use J3 and the dataset named “GAA\_pset.csv” to examine the entire 10 objective Pareto set. An executive at the firm would like an aircraft that meet the following criteria:

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Objective Name:** | NOISE | WEMP | DOC | ROUGH | WFUEL | PURCH | RANGE | LDMAX | VCMAX | PFPF |
| **Criteria** | <73.6 | <1960 | <68.9 | <1.9 | <423 | <43,250 | >2217 | >15.4 | >195.1 | <1 |

How many aircraft meet these criteria? What would you advise the executive to do? (3 points)

1. Your clients are interested in releasing three models of aircraft for different market segments, an economy model, that has low operating cost (DOC) and a low purchase price (PURCH); a comfort model that has low NOISE and low roughness (ROUGH); and a performance model that has high RANGE, high LDMAX and high VCMAX. They would like all airplanes to have low PFPF to facilitate the manufacturing process.

Make a set of visualizations that narrate a recommended set of options for each model (i.e. show relevant trade-offs and highlight important aspects of the set). Write a brief paragraph for each that explains each figure and outlines relevant trade-offs and opportunities (15 points).

1. Design your own aircraft and use visualizations to explain what features you are highlighting and how you navigated trade-offs between objectives. This is meant to be open-ended, there is no correct answer, but you must explain your logic and narrate your design with visualizations (5 points).

Bonus

# Sensitivity Analysis

In this problem we’ll explore sensitivity analysis as a tool to identify the inputs to a model that cause the largest changes in a particular output of interest. Imagine that you’re consulting for a company that manufactures widgets. There are three characteristics that define a widget that consumers will potentially care about: its height, weight, and color. The company’s engineers have identified a regression relationship that approximates the profit per widget as a function of these three characteristics:

where *x*1, *x*2, and *x*3 are quantitative representations of the height, weight, and color that have all been transformed to the unitless range (-π,π). The coefficients are: *a* = 7 and *b* = 0.1.

Due to budget constraints, the company will only be able to change one of these characteristics on the manufacturing line. It is up to you to determine which factor has the most influence on the profit.

**Part A: One-at-a-time (OAT) sensitivity analysis (1 quiz point)**

In a programming language of your choice (R, Matlab, Python, etc., not Excel), write the equation above as a function that accepts a vector of three input variables and returns the profit estimate *y*. Use this function to perform a one-at-a-time sensitivity analysis. This is essentially a finite difference estimate, where the sensitivity of each factor *x*i is quantified by *Si* = (∆*y*)/(∆*xi*) for some small ∆*xi*. If the initial point is

(*x*1,*x*2,*x*3) = (1.0,1.0,1.0), find all three sensitivity indices using ∆*xi* = 0.01 for each variable.

**Please submit a hard copy your code from your .py, .m, .r, or similar file (screenshots of code or output are not acceptable) along with your estimates of the three sensitivity indices.** Which variable appears to be the most influential?

**Part B: Global sensitivity analysis (3 quiz points)**

The OAT analysis you performed in Part A is known as a local sensitivity analysis, because it is centered around a particular point. By contrast, a global sensitivity analysis samples the input variables across their entire range, which in this case is (-π,π). One commonly used global method is Sobol sensitivity analysis, which aims to estimate the fraction of variance in the output that each input factor contributes.

Because the Sobol method is a bit more complicated to code, it is recommended that you download a third-party library for this part of the assignment. Being able to locate and understand other people’s software is a time-saving skill, particularly when the software in question is well documented and tested. You are welcome to use any implementation of the Sobol method that you can find; here are a few:

**Matlab**: A version of the Sobol method is provided in the SAFE toolbox for sensitivity analysis. You can download here: http://bristol.ac.uk/cabot/resources/safe-toolbox/

**Python**: An open-source sensitivity analysis library is available here:

<http://jdherman.github.io/SALib/>

The examples/ directory shows a few different ways to run the method (either from a Python script or from the command line). There is also a blog post that you can read describing the steps in detail:

<http://waterprogramming.wordpress.com/2013/08/05/running-sobol-sensitivity-analysis-using-salib/>

**R**: An open source sensitivity analysis library in R is available here:

<http://cran.r-project.org/web/packages/sensitivity/index.html>

It comes with a detailed PDF manual showing how to call the different functions. This library contains several different implementations of the Sobol method; any of them should do for our application.

Regardless of which library you choose, there are typically three steps to calculating the Sobol sensitivity indices. You will need to read the code and/or documentation for your library of choice to figure out how to perform these steps:

1. Create a sample of the input space. Choosing a sample size requires some guesswork, but for this problem *N* = 1000 should provide accurate results.
2. Evaluate the model for all of the input samples. Here you can use the function you created in Part 1, or create a new one if you switched languages.
3. Use the model output to calculate the sensitivity indices.

The Sobol method, depending on your version, should return both first-order and total-order sensitivity indices. First-order indices represent the sensitivity of each factor by itself, while total-order indices account for interactions with other parameters.

Report the first-order and total-order Sobol indices that you calculate in your experiment. You do not have to submit the code for the library that you used, but include any additional code ***you*** wrote (hard copy of .py, .m, .r, or similar file, no screenshots), or at least describe the sequence of steps that you followed to perform the calculation.

**Part C: Decision implications (1 quiz point)**

This is a simplified example, but it represents a problem that comes up frequently: given a set of uncertain inputs and a limited budget, which of them matter enough to warrant further research? Provide a brief discussion comparing the sensitivity indices that you found in Part A (local OAT) and Part B (total-order global sensitivity). Do they provide the same ranking of importance? If so, why? If not, why not? Is there anything about the mathematical form of the function that would make the results similar or different?

What about the first-order indices from Part B—do these provide the same ranking as the local OAT sensitivity indices from Part A? Why or why not?

Briefly discuss the consequences of using OAT rather than global sensitivity analysis for this problem. Do you think this is a generalizable result? Explain.