ECMT 3160: Assignment 1

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1   Instructions

- There are 3 questions in this assignment. Answer all questions.

- The marks awarded to each part of a question are indicated.

- You must provide explanations and reasoning for all of your answers in order to receive marks. Answers without justification and explanation receive a mark of zero.

- This is an individual assignment – i.e., not group work. You must hand in your own work. This means writing up your answers in your own words and reasoning. Failure to do so is a breach of integrity, and will be sent to the disciplinary board for further action.

2   Questions

Question 1. *(4 Marks in total).* Show that $A = \emptyset$ if and only if $B = A \Delta B$.
Hint: You may use carefully labelled Venn Diagrams to help explain your reasoning, but on their own, they are insufficient to get full marks.

Question 2. *(12 Marks in total, 3 marks per part).* You are a member of the Rebel Alliance in the Star Wars Universe who is trapped on the planet Tatooine, and are being pursued by the forces of the evil Empire. To escape the Empire, you have to find the revered Jedi master Obi Won Kenobi, as he can arrange safe passage for you to leave the planet to return to your base. To evade capture, you hide in a cantina in the city of Mos Eisely. Outlaws make up three-quarters of the cantina's clientele. If you ask a law-abiding person for directions to Obi Won, the answer is correct with probability 2/3. If you ask an outlaw for directions to Obi Won, the answer is correct with probability $1/2$. Answers to repeated questions are independent even if the question is the same. You cannot tell whether the person you ask is an outlaw or is law-abiding, and all you know is that Obi Won resides in the North or South of Mos Eisely.

1. You ask one of the persons in the cantina whether Obi Won is South or North of Mos Eisely. The answer is South. What is the probability this is correct?

2. You ask the same person again, and receive the same reply. What is the probability that it is correct?

3. You ask the same person one more time, and receive the same reply. What is the probability that it is correct?

4. You ask the same person a fourth time, and receive the same answer. What is the probability that it is correct?

Question 3. *(5 Marks in total).* A lottery selects r numbers from the first n integers. Show that the probability that all r numbers has at least k spacers between each pair of them is

$$\frac{\binom{n-(r-1)k}{r}}{\binom{n}{r}}, \quad (r-1)k \leq n.$$