1. For logistic regression with one predictor, we use the model

$$\log\left(\frac{\theta(x)}{1 - \theta(x)}\right) = \beta_1 + \beta_1 x$$

(a) Show that solving for the probability of success for a given value of the predictor, $\theta(x)$, gives

$$\theta(x) = \frac{\exp\left(\beta_0 + \beta_1 x\right)}{1 + \exp\left(\beta_0 + \beta_1 x\right)}$$

(b) and

$$\theta(x) = \frac{1}{1 + \exp\left(-\left\{\beta_0 + \beta_1 x\right\}\right)}$$

2. On page 285 of the text, it says "When $X$ is a dummy variable, it can be shown that the log odds are also a linear function of $x$." Suppose that $X$ is a dummy variable, taking the value 1 with probability $\pi_j$, $j = 0, 1$, conditional on $Y = 0, 1$.

   (a) Show that the log odds are a linear function of $x$.

   (b) Define the slope and intercept for the linear function.

3. On page 284 of the text, the author quotes Cook and Weisberg: "When conducting a binary regression with a skewed predictor, it is often easiest to assess the need for $x$ and $\log(x)$ by including them both in the model so that their relative contributions can be assessed directly." Show that indeed the log odds are a function of $x$ and $\log(x)$ for the gamma distribution.

4. Chapter 8, Question 4