Ba IP plan
Data-driven control of polynomial systems via SOS

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The increased availability of data has incentivized the development of controller-design techniques based only on data, rather than on a model. These techniques focus mainly on simple linear dynamical systems like

$$\dot{x} = Ax + Bu. \tag{1}$$

Instead, to capture a larger spectrum of behaviors, we consider here the polynomial dynamical system

$$\dot{x} = f(x) + g(x)u. \tag{2}$$

Also here we are interested in designing a state-feedback controller from data, so the polynomial vector fields f and g are unknown to the controller-design algorithm.

It is possible to write a polynomial system in a linear-like form as

$$\dot{x} = AZ(x) + BW(x)u \text{ with } f(x) = AZ(x) \text{ and } g(x) = BW(x), \tag{3}$$

where $Z(x)$ and $W(x)$ are known matrices of monomials and A, B are unknown matrices of coefficients.

Example: For the polynomial $f(x) = x + 2xy + 3y$, we have $Z(x) = \begin{bmatrix} x & y & xy \end{bmatrix}^\top$ and $A = \begin{bmatrix} 1 & 3 & 2 \end{bmatrix}$.

The thesis aims at numerically verifying the feasibility of data-based control design for polynomial systems through sum-of-squares programs [1].

The design is applied to the automatic steering of a ship. A (relevant) part of the ship dynamics is captured by

$$\dot{r} = -\frac{\alpha_3}{T} r^3 - \frac{\alpha_1}{T} r + \frac{J}{T} u \tag{4}$$

where T is a time constant, r is the yaw angle (corresponding to the state variable to control), α_1 and α_3 are the Norrbin's coefficients, and J is a gain constant. More information on this model can be found in [3, Section 2].

The project consists of the following steps.

1. Write the Norrbin model as a polynomial system and simulate it with MATLAB to generate the data points for the design of a data-driven controller.

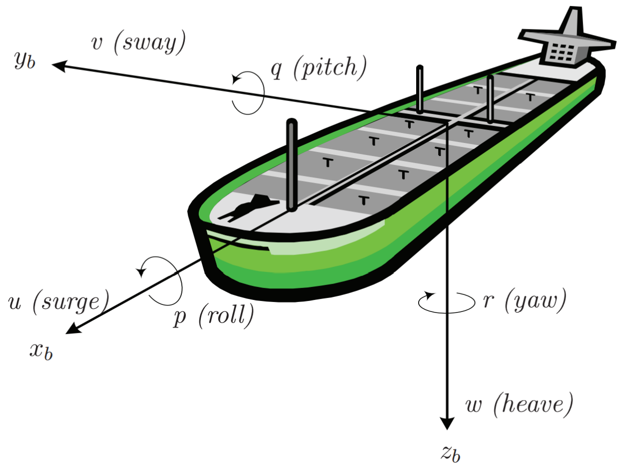

Figure 1: Motion variables for a marine vessel [2].

2. Obtain for this simple polynomial system the quantities and conditions of [4, Theorem 2]. In a nutshell, [4, Theorem 2] shows how to design a rational state-feedback control law $u(x) := \frac{p(x)}{q(x)}$ for a set of system parameters compatible with data.

3. Get familiar with SOS programs and how to implement them. For the former task (understand what they are), [5] seems the best starting point; [1] is an excellent reference as well, but possibly complex. For the latter task (how to implement them), the following toolbox and software needs to be installed.

   - MATLAB - https://it.mathworks.com/help/matlab/getting-started-with-matlab.html
   - Yalmip - https://yalmip.github.io/tutorial/installation/
   - Solver - https://yalmip.github.io/solver/mosek/. Download the file and request an academic license (https://www.mosek.com/license/request/?i=acp)

4. The key step of the project is implementing [4, Algorithm 1] for control design. [4, Algorithm 1] is a SOS relaxation of [4, Theorem 2]. Start taking a look at the algorithm and understand as much as possible.

5. Implement [4, Algorithm 1] in MATLAB through YALMIP. Examples and hints in [4, Section IV] can come in handy. Verify the performance of the algorithm.

6. Depending on the student's interest and the actual progress of the project, the same implementation can be used for an alternative, more natural stabilization condition.

References

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