

## ECE 60000 Random Variables and Signals – Spring 2020

### Homework 11

Due date: Wednesday, April 29, 2020

1. (10 points) Consider an M/M/1 queueing system with service rate of two customers per second.
  - (a) (5 pts) Find the maximum allowable arrival rate if 90% of customers should not have a delay of more than 3 seconds.
  - (b) (5 pts) Under such case (maximum arrival rate in (a)), what is the average queue length?
  
2. (20 points) Consider an M/M/1 system where the customer arrival rate is  $\lambda$ , the service times the server is an exponential random variable having mean  $1/\mu$ , and the customers are impatient. Specifically, upon arrival, customers check the number of customers in the system and join the queue with probability  $1/(n + 1)$  (or equivalently, leave with probability  $n/(n + 1)$ ), where  $n$  is the number of customers in the system.
  - (a) (5 pts) Draw the transition rate diagram.
  - (b) (5 pts) Find the limiting probability  $p_n$  of finding  $n$  in the system,  $n \geq 0$ .
  - (c) (5 pts) Find the average number of customers in the system.  
*Hint: Use the math fact  $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$*
  - (d) (5 pts) Find the average amount of time a customer spends in the system.
  
3. (20 pts) Inquiries arrive at an information center according to a Poisson process of rate 2 inquiries per second. It takes a server an exponential amount of time with mean  $\frac{1}{2}$  second to answer each query.
  - (a) (5 pts) If there are two servers. What is the probability that an arriving customer must wait to be served?
  - (b) (5 pts) Find the mean number of customers in the system and the mean time spent in the system.
  - (c) (10 pts) If the arrival rate increases to 10 inquiries per second and the number of servers is increased to 6, what is the resulting probability that an arriving customer finds all servers busy? What is the mean total delay for each inquiry? What is the percentage of all queries with waiting time less than 1 second?