

# ECO-6004B Economics of Alternative Investments

## Lecture 04: Commodities: Applications and Evidence.

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## The Road-map of the module

- **Week 01:** An introduction to Alternative Investments.
- **Week 02:** Measures of Risk and Performance.
- **Weeks 03–04: Managed futures portfolio trading strategies.**
- **Weeks 05–06:** Real Estate.
- **Week 07:** Hedge Fund Investment Program.
- **Week 08:** Private Equity.
- **Week 09:** Investment in Structured Products.
- **Week 10:** Dynamic Portfolio Management.
- **Week 11:** Module Review.

## Readings for Lecture 04

- Alternative Investments - CAIA Level I.

Chapter 12: Commodities: Applications and Evidence.

- Handbook of Alternative Assets.

Chapter 12: Introduction to Commodities.

Chapter 13: Investing in Commodity Futures.

Chapter 14: Commodity futures in a Portfolio Context.

Chapter 15: Managed Futures.

## Outline of Lecture 04

- Recap on Portfolio Theory.
- Commodity Investing for Diversification.
- Commodity Investing for Return Enhancement.
- Investing in Commodities without Futures.
- Commodity Exposure Through Futures Contracts.
- Commodity Risks and Returns.

# Recap on Portfolio Theory

- By spreading investment over more than 1 security, portfolio's expected return is the weighted average return of the securities held in the portfolio.
- There is a change of losing all the capital, if it is all invested in just one type of security.
- By combining securities with less than perfect correlation, investors can reduce the risk of the portfolio through diversification.

## Recap on Portfolio Theory

- The effectiveness of this reduction will depend on the **degree of correlation** between the movements of the security returns.
  - ✓ If two securities' returns are perfectly positively correlated, they move up and down together in proportion. If they were combined in a portfolio, the investor would not get any risk reduction.
  - ✓ If two securities' returns are perfectly negatively correlated, then they move up and down in exact opposition and in proportion. If the invest was to combine two such securities in a portfolio, he/she would eliminate risk entirely!

In practice, negative correlation or low correlated assets are not easy to find. Presuming less than perfect correlation, diversification reduces risk.

# Recap on Portfolio Theory

- We can now consider the formula for computing the risk of a portfolio (variance) that invests in  $N$  assets.
- The variance of a portfolio is the weighted average of the individual variances ( $\omega_i^2$ ), where the weights  $\omega_i$  are squared, plus the weighted covariances ( $\sigma_{i,j}$ ) between all the pairs of individual assets in the portfolio.

$$\text{Var}(R_P) = \sigma_P^2 = \sum_{i=1}^N \omega_i^2 \sigma_i^2 + \sum_{i \neq j=1}^N \omega_i \omega_j \sigma_{i,j}$$

or

$$\sigma_P^2 = \sum_{i,j=1}^N \omega_i \omega_j \sigma_{i,j} = \sum_{i,j=1}^N \omega_i \omega_j \rho_{i,j} \sigma_i \sigma_j$$

where  $\rho_{i,j}$  is the correlation coefficient between the returns of assets  $i$  and  $j$ .

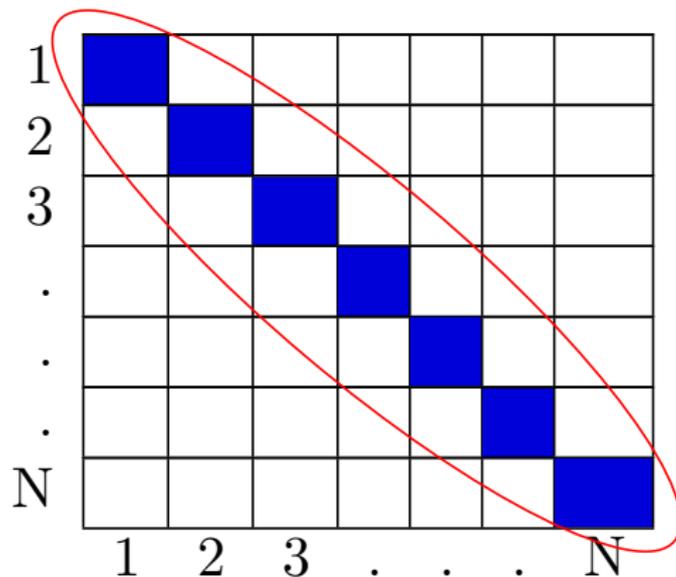
# Recap on Portfolio Theory

The risk of a 2-assets portfolio (variance of portfolio) is the sum of all the elements of the following matrix:

	Stock 1	Stock 2
Stock 1	$\omega_1^2 \sigma_1^2$	$\omega_1 \omega_2 \sigma_{1,2} = \omega_1 \omega_2 \rho_{1,2} \sigma_1 \sigma_2$
Stock 2	$\omega_2 \omega_1 \sigma_{2,1} = \omega_2 \omega_1 \rho_{2,1} \sigma_2 \sigma_1$	$\omega_2^2 \sigma_2^2$

where  $\omega_i$  is portfolio weight in asset  $i$ ,  $\sigma_i$  is standard deviation of the return on asset  $i$ ,  $\sigma_{i,j}$  is covariance of the returns of assets  $i$  and  $j$ , and  $\rho_{i,j}$  is the correlation of the returns of assets  $i$  and  $j$ .

# Recap on Portfolio Theory



To calculate portfolio variance sum the values in the cells.

The shaded cells contain variances, while the remainder contains covariances.

## Recap on Portfolio Theory

Number of assets	Number of variances	Number of covariances	Assume an equal amount in each asset
2	2	2	1/2
3	3	6	1/3
4	4	12	1/4
10	10	90	1/10
100	100	9900	1/100
N	N	$N^2 - N$	1/N

## The importance of covariances

- Total variance of a portfolio of  $N$  assets, when  $N$  is large is equal to the average covariance between the  $N$  assets that make up the portfolio.
- A very important conclusion is that when we are dealing with large portfolios, the risk (variance) of an individual asset  $i$  we are thinking of adding to the portfolio is not important.
- What is truly important is the covariance between it and the assets already in the portfolio.

# Portfolio Construction

## How do investors construct portfolios?

There are many ways, but two types of portfolios need to be considered:

- i. Minimum Variance portfolio;
- ii. Maximum Sharpe Ratio portfolio.

## Minimum Variance Portfolio

- One particular portfolio of interest is the portfolio with the lowest risk – the minimum variance portfolio.
- Subject to the assumption of total investment of the capital, i.e.  $\omega_1 + \omega_2 = 1$ , we have

$$\sigma_P^2 = \omega_1^2 \sigma_1^2 + (1 - \omega_1)^2 \sigma_2^2 + 2\omega_1(1 - \omega_1)\sigma_{1,2}$$

- Differentiate both sides with respect to  $\omega_1$  to obtain:

$$\begin{aligned} \frac{\partial \sigma_P^2}{\partial \omega_1} &= 2\omega_1 \sigma_1^2 - 2(1 - \omega_1) \sigma_2^2 + 2(1 - 2\omega_1) \sigma_{1,2} \\ &= 2\omega_1 (\sigma_1^2 + \sigma_2^2) - 2\sigma_2^2 + 2\sigma_{1,2} - 4\omega_1 \sigma_{1,2} \\ &= 2\omega_1 [\sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}] - 2\sigma_2^2 + 2\sigma_{1,2} \end{aligned}$$

# Minimum Variance Portfolio

- The first order condition for finding the minimum is to set the derivative to zero, i.e.

$$\frac{\partial \sigma_P^2}{\partial \omega_1} = 0$$

- This yields the following equation for  $\omega_1^*$ , where the ‘\*’ denotes an optimal value:

$$\omega_2^* = \frac{\sigma_2^2 - \sigma_{1,2}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_{1,2}}$$

# Maximum Sharpe Ratio Portfolio

- To find the Maximum Sharpe Ratio Portfolio, we need to assume that there is a risk-free asset available.
- Then we solve the following optimization problem:

$$\max_{\omega_1, \omega_2} \theta = \frac{\mu_P - r_F}{\sigma_P},$$

subject to

$$\omega_1 + \omega_2 = 1,$$

where  $\mu_P$  is the expected return of the optimal portfolio,  $r_F$  is the risk-free rate, and  $\sigma_P$  is the standard deviation of the portfolio returns.

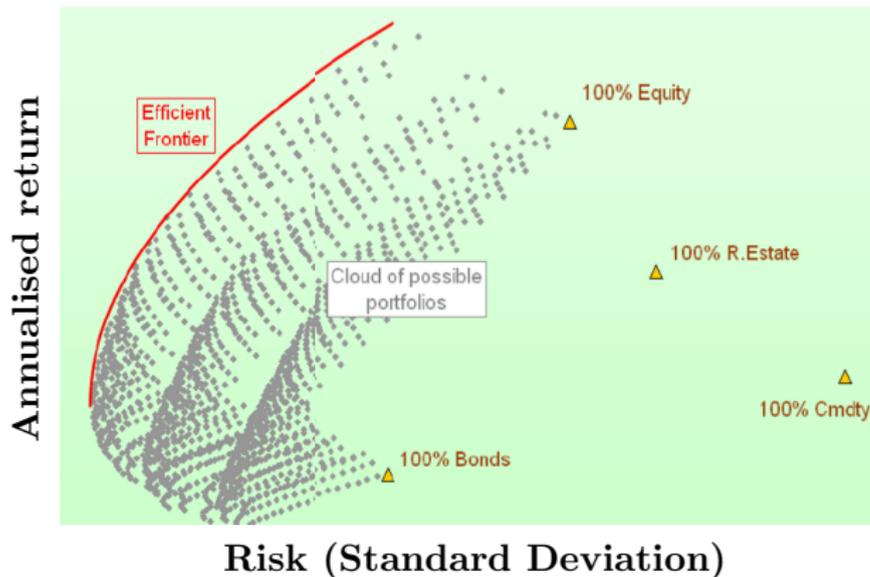
## A Technique for Finding the Efficient Frontier

- We need identify 2 **frontier portfolios**, and then find convex combinations of them.
- To find the initial two portfolios, we can maximise the Sharpe Ratio, assuming each time a different risk-free rate.
- A simple combination rule to find other points of the efficient frontier is the following:
  - Given any two weight vectors  $\mathbf{x} = [x_1 \dots x_n]$  and  $\mathbf{y} = [y_1 \dots y_n]$ , all frontier portfolios are formed as convex combinations of  $\mathbf{x}$  and  $\mathbf{y}$ .
  - In other words, a third portfolio  $\mathbf{z}$  has weights given by

$$\mathbf{z} = [\alpha x_1 + (1 - \alpha)y_1 \dots \alpha x_n + (1 - \alpha)y_n],$$

where  $\alpha$  is an arbitrary constant.

## Example of a Efficient Frontier



The **efficient frontier** represents the set of optimal portfolios that offer the highest expected return for a defined level of risk or the lowest level of risk for a given level of expected return.

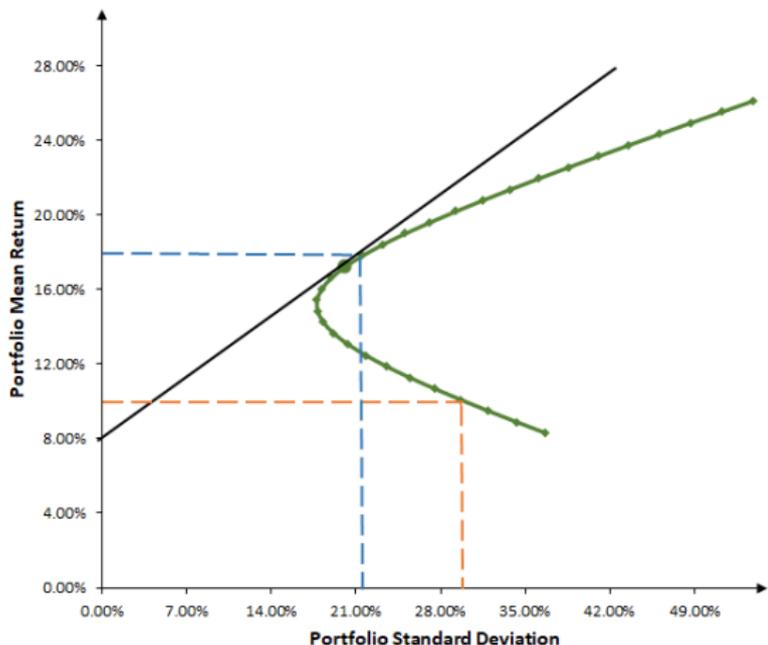
## Recap on Portfolio Theory



Stocks offer an expected rate of return of 18%, with a standard deviation of 22%. Gold offers an expected return of 10% with a standard deviation of 30%. For simplicity, assume the covariance to be equal to zero.

In light of the apparent inferiority of gold with respect to both return and risk, would anyone hold gold?

# Recap on Portfolio Theory



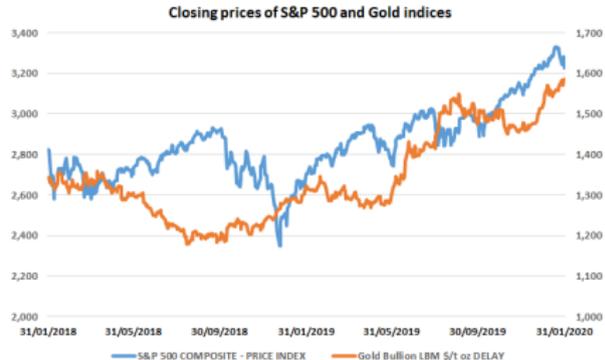
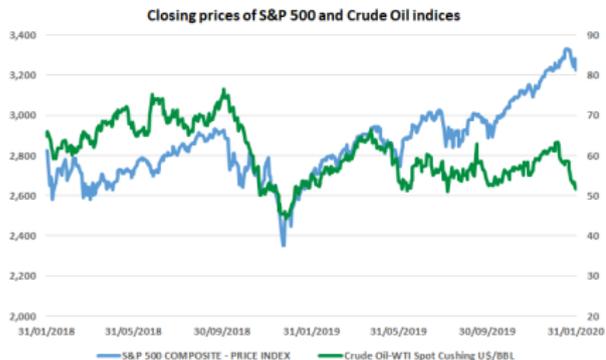
Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a part of a portfolio if the correlation between gold and stocks is sufficiently low.

# Commodity Investing for Diversification

Commodity returns may have low correlation with traditional assets (stocks and bonds):

- Commodity prices are not directly determined by the discount value of future cash flows, but driven by forecasts of supply and demand.
- Nominal commodity prices are positively correlated with inflation while prices of stock and bonds tend to be negatively correlated with inflation because inflation raises the discount rates.
- Commodity prices react differently at different parts of business cycle and depend on current economic conditions and factors relating to short term supply and demand.
- Commodities represent a major cost of some corporate producers, hence negatively correlating with corporate profits.

# Commodity Investing for Diversification



	S&P 500	Gold	Crude Oil	MSCI EM
S&P 500	1.0000			
Gold	-0.0952	1.0000		
Crude Oil	0.2306	0.0422	1.0000	
MSCI EM	0.4458	-0.0104	0.2368	1.0000

## Commodity as Diversifier in Perfect Market Equilibrium

- In a perfect market equilibrium, the perfectly diversified portfolio is the market portfolio that contains exposure to all assets.
- The percentage of the total market portfolio attribute to each asset is known as market weight.
- The market weight of an asset is equal to the percentage of total global value of that asset relative to the total global value of all assets.
- For example, if copper represents 4.6% of total wealth of the world, then a perfect diversified portfolio of risky assets should have an 4.6% weight in copper.
- Commodities are a substantial part of total wealth, but the issue is to determine the weight of that commodity.

## Commodity as Diversifier in the Presence of Market Imperfections

- In practice, market is imperfect and may remain out of equilibrium for extended period of time (e.g. shortages or oversupplies may last a longer time).
- Investors seek to hold commodities in the proportion that provides the highest return-to-risk ratio based on their existing portfolios rather than market weight.
- Empirical evidence can be a tool for ascertaining the benefits of diversification for each commodity to each investor.

# Commodity Investing for Return Enhancement

## Return Enhancement: Alpha

- Market participants speculate on idiosyncratic movements in the underlying commodity prices.
- Investors use technical and fundamental analysis to forecast commodity prices and to identify trades with superior risk-adjusted returns.
- Example: managed futures funds use technical analysis to forecast trends in natural gas prices resulting from trading activity or seasonal patterns.

# Commodity Investing for Return Enhancement

## Return Enhancement: Beta

- In perfect equilibrium, expected returns from assets depend solely on the amount of systematic risk.
- Commodities do not enhance expected returns when they are efficiently priced and when their systematic risk (betas) are low.
- Return enhancement from beta must be attributable to market inefficiencies or markets in disequilibrium.

# Investing in Commodities without Futures

- Investing in Physical Commodities.
- Investing in Commodity-Related Equities.
- Exchanged-Traded Funds (ETF).
- Commodity-Link Notes (CLN) .

## Investing in Physical Commodities

- Investors can purchase and physically hold an underlying commodity to gain economic exposure to commodity return.
- Physical ownership of commodities offers benefits of convenience yield but also costs of storage and transportation.
- Examples: Gold are hold as “safe-haven”, natural gas inventory buildup for production because of its seasonal nature.

## Investing in Commodity-Related Equities

- Investors can own the securities of a firm that derives a substantial part of its revenues from the sale of physical commodities such as natural resource company.
- Not potential for diversification purpose because:
  - Most firms have revenue related to a variety of commodities.
  - High correlation between stock price and commodity price assumes that the firm has not hedged its exposure in forward or futures contract.
  - Commodity equities have two betas: one relates to commodity market and a second to the equity market.
  - Financial and operating leverage may vary and affect return of an investment in the way that is uncorrelated with commodity prices.

## Exchanged-Traded Funds (ETF)

- ETF is one of the easiest ways to invest in a basket of commodities or in some individual commodities (gold, silver, oil, ...).
- Commodity ETFs may either hold the actual commodity, or purchase futures contracts, or invest in the equity securities of commodity-producing firms.
- Popular commodity ETFs: United States Oil Fund (USO), SPDR Gold Trust (GLD), PowerShares DB Commodity Index Tracking Fund (DBC), GSG—the iShares S&P GSCI Commodity-Indexed Trust ETF, ...

## Commodity-Linked Notes (CLN)

- A Commodity-Linked Note is an intermediate term debt instrument whose value at maturity is a function of the value of an underlying commodity or basket of commodity.
- A gold-mining firm can issue a Commodity-Linked Note that has coupon or principal payments directly linked to the price of gold.
- Commodity-Linked Notes are closely linked to commodity prices as well as idiosyncratic default of issued firms.
- Commodity-Linked Notes are attractive for investors with restrictions on direct position in futures contract.

# Commodity Exposure Through Futures Contracts

The Return on a Futures Contract Differs from Spot Return due to basis risk.

The Return on a fully collateralized position in a futures contract is:

$$R_{f\ coll} = \text{Spot Return} + \text{Collateral Yield} + \text{Roll Yield}$$

where the **spot return** is the return on the underlying asset in the spot market, the **collateral yield** is the interest earned from riskless bonds used to collateralize the futures contract, and the **roll yield** is the portion of the return of futures position from the changes in its basis.

# Commodity Exposure Through Futures Contracts

We have learned from previous lecture, forward price of a physical asset is given by:

$$F(T) = e^{(r+c-y)T} S$$

where  $F$  is forward price,  $S$  is spot price, and  $T$  denotes the time period of forward contract,  $r$  is interest,  $c$  is storage cost,  $y$  is convenience yield.

## Example

Consider a 1-year futures contract on a physical asset that is traded at the spot price of \$25 per unit. The storage cost is 2% per year, annual interest rate is 5% and convenience yield is 1%. This corresponds to  $c = 2\%$ ,  $r = 5\%$ ,  $y = 1\%$  and  $S = 25$ ,  $T = 1$ .

The theoretical futures price should be:

$$F(T) = \$25 \cdot e^{(5\%+2\%-1\%)1} \approx \$26.55$$

How an arbitrageur makes profit:

- (i) if the actual futures price is greater than 26.5459?
- (ii) if the actual futures price is lower than 26.5459?

## Scenario (i)

$$F(T) > e^{(r+c-y)T} S$$

*If the actual price is greater than 26.5459, an arbitrageur can buy the asset and short 1-year futures contracts to lock in a profit.*

As arbitrageurs do so, there will be a tendency for  $S$  to increase and  $F$  to decrease until  $F(T) = e^{(r+c-y)T} S$ .

## Scenario (ii)

$$F(T) < e^{(r+c-y)T} S$$

*If the actual price is less than 26.5459, an investor who already owns the asset can improve the return by selling the asset and buying futures contracts.*

## Scenario (ii)

$$F(T) < e^{(r+c-y)T} S$$

*If the actual price is less than 26.5459, an investor who already owns the asset can improve the return by selling the asset and buying futures contracts.*

This strategy may not be applicable for a commodity that is a consumption asset because individuals and companies who own a consumption commodity usually plan to use it in some way. There is therefore nothing to stop such inequality.

Forward price should be expressed thus:

$$F(T) \leq e^{(r+c-y)T} S$$

## Other considerations

- The convenience yields and storage costs of market participants may differ and are unobservable.
- If  $r + c - y = 0$ , futures price equals spot price.
- The term structure of forward price will be upward or downward sloping, depending on the the expected value of these three parameters.
- If the term structure is informationally inefficient, arbitrageurs may use available information to take a long position in under-priced asset and short position in overpriced contract.

# Commodity Risks and Returns

There are four main Commodity Market Event Risk Attributes:

- Most global events cause increase in commodity prices due to anticipated decrease in commodity supplies or increases in demand (trade wars, military wars, major weather events, political instability).
- Commodity price increases tend to be larger and more sudden than price decrease, providing long positions in commodities with positively skewed returns.
- Many commodity shocks are likely to be uncorrelated with each other.
- Shocks to commodity markets are generally uncorrelated with financial markets.

# Commodity Risks and Returns

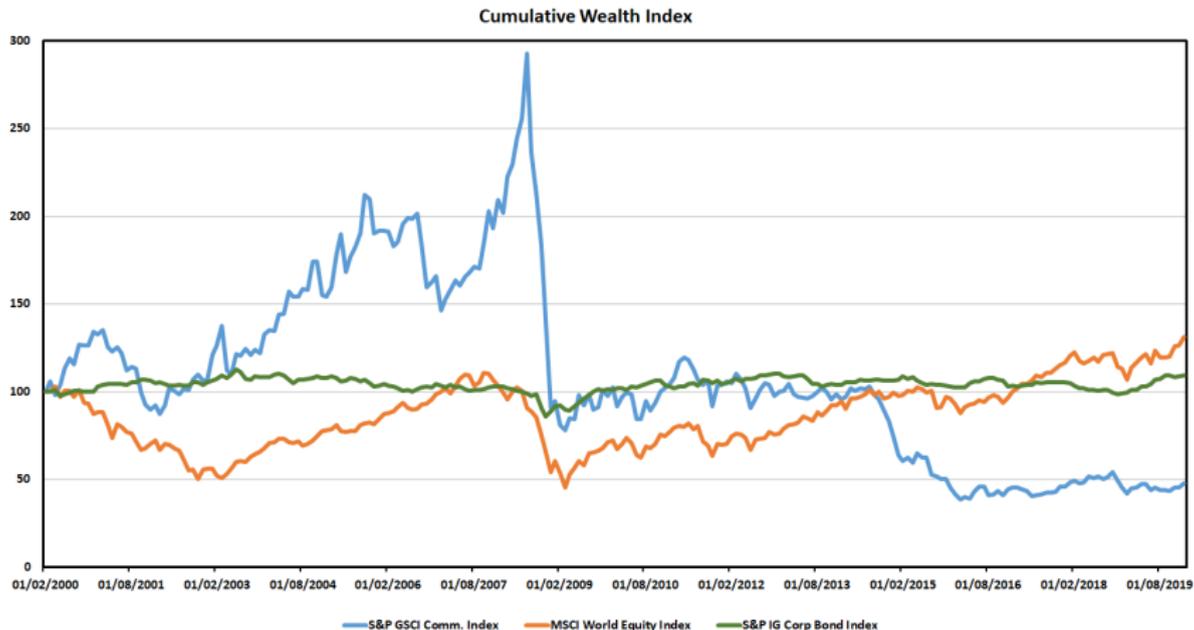
Commodities may be viewed as a defensive investment:

- Traditional assets do not provide downside risk protection: global equity markets have high correlations during period of market stress.
- Most traditional investments do not offer both protection from global turmoil and attractive returns.
- Hedge funds and other skilled-based strategies may provide diversification and having returns protected from market turmoil.

# Historical Risks and Returns

<b>Index (Mar.00-Jan.20)</b>	S&P GSCI Comm. Index	MSCI World Equity Index	US T-Bond 10Y Index	S&P IG Corp Bond Index	S&P HY Corp Bond Index	US T-Bond 3M Index
Ann. Mean	-0.746%	2.698%	3.388%	0.171%	25.761%	0.136%
Ann. Std. Dev.	23.572%	16.210%	0.351%	1.219%	3.443%	0.517%
Skewness	-0.974	-0.495	0.276	1.744	2.706	1.509
Kurtosis	3.058	1.943	-0.925	5.016	10.583	-0.019
Range	51.379%	34.496%	0.415%	2.318%	7.270%	0.531%
Minimum	-35.570%	-17.654%	0.122%	-0.579%	0.416%	0.000%
Maximum	15.809%	16.842%	0.536%	1.739%	7.686%	0.531%
Max Drawdown	-81.549%	-56.183%	-0.412%	-4.328%	-12.179%	-0.528%
Sharpe Ratio	-0.029	0.019	1.446	-0.346	2.023	0.000
Alpha	-0.001	-0.001	0.000	-0.003	0.014	0.000
Beta	-0.682	1.265	1.000	1.066	4.012	0.000
Autocorrelation	0.092	-0.005	0.992	0.947	0.994	0.985
N. of Obs.	239	239	239	239	239	239

## Historical Risks and Returns



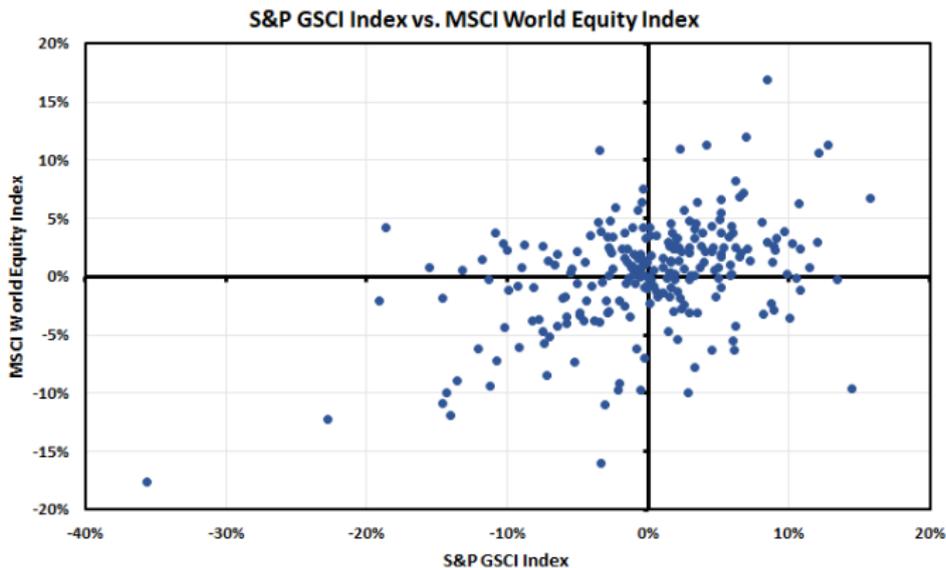
# Betas and correlations

<b>Multivariate Betas</b>	MSCI World Equity Index	S&P IG Corp Bond Index	US T-Bond 3M Index	US T-Bond 10Y Index	Intercept	Adj-R <sup>2</sup>
S&P GSCI Comm. Index	0.603***	-3.040**	0.319	13.233**	-0.039***	0.216

<b>Univariate Betas</b>	MSCI World Equity Index	S&P IG Corp Bond Index	US T-Bond 3M Index	US T-Bond 10Y Index
S&P GSCI Comm. Index	0.631*	-3.312	4.172	7.366**

<b>Correlation matrix</b>	S&P GSCI Comm. Index	MSCI World Equity Index	S&P IG Corp Bond Index	US T-Bond 3M Index	US T-Bond 10Y Index
S&P GSCI Comm. Index	1.000				
MSCI World Equity Index	0.434***	1.000			
S&P IG Corp Bond Index	-0.171***	-0.222***	1.000		
US T-Bond 3M Index	0.091	-0.054	0.245***	1.000	
US T-Bond 10Y Index	0.110	-0.077	0.387***	0.761***	1.000

# Scatter Plot of the Returns



# Why investing in commodities can be challenging?

**Why investing in commodities can be challenging?**