Statistical Computing

Homework #1

Due Date: Monday Feb 1

Notes about your assignment: You should use R to answer each of these questions. In your R script please label the questions in order using comments (as well as any sub-parts), and include the answer to each question (typically, what you get when you run your code) as comments at the end of each question.

1. If $x_0 = 5$ and
$$x_n = 3x_{n-1} \bmod 150$$
find $x_1, ..., x_{10}$

2. If $x_0 = 3$ and
$$x_n = (5x_{n-1} + 7) \bmod 200$$
find $x_1, ..., x_{10}$

For questions 3-9, use Monte Carlo integration to approximate the given integrals. If you are able to calculate the integral analytically, you may want to use this to double-check your work!

3. $\int_0^1 e^{e^x} dx$
4. $\int_0^1 (1 - x^2)^{\frac{3}{2}} dx$
5. $\int_{-2}^2 e^{x+x^2} dx$
6. $\int_0^\infty x(1 + x^2)^{-2} dx$
7. $\int_{-\infty}^\infty e^{x^2} dx$ [Hint: symmetry is your friend!]
8. $\int_0^1 \int_0^1 e^{(x+y)^2} dydx$
9. $\int_0^\infty \int_0^x e^{-(x+y)} dydx$

   [Hint: Let $I_Y(x) = \begin{cases} 1 & y < x \\ 0 & y \geq x \end{cases}$. Use this function to equate the integral to one in which both terms go from 0 to ∞]

10. Use simulation to approximate $Cov(U, e^U)$ where U is uniform on $(0, 1)$.
11. Let U be uniform on $(0, 1)$. Use simulation to approximate the following:
    a) $Cov(U, \sqrt{1 - U^2})$
    b) $Cov(U^2, \sqrt{1 - U^2})$
12. For uniform $(0, 1)$ random variables $U_1, U_2, ...$ define N to be the number of random numbers that must be summed to exceed 1:

    $N = \text{Minimum}\{n : \sum_{i=1}^n U_i > 1\}$

    a) Estimate $\mathbb{E}\{N\}$ by generating 100 values of N

b) Estimate $\mathbb{E}\{N\}$ by generating 1,000 values of N

c) Estimate $\mathbb{E}\{N\}$ by generating 10,000 values of N

d) Can you guess the true value of $\mathbb{E}\{N\}$?

13. For uniform $(0, 1)$ random variables, define N to be the maximum number of random numbers whose product is still at least e^{-3}:

    $N = \text{Maximum}\{n : \prod_{i=1}^{n} U_i \geq e^{-3}\}$

    a) Find $\mathbb{E}\{N\}$ by simulation

    b) Find $P(N = i)$ for $i = 0, 1, 2, 3, 4, 5, 6$ by simulation