

HW Week 9: Due Wednesday Apr 7 by 11:59 pm, via Canvas

March 27, 2021

1. I toss a coin 100 times. The coin is fair (it has equal probability of coming up heads or tails). Use R to find the probability that:
 - (a) I get exactly 50 heads (hint: `dbinom()`)
 - (b) I get at most 50 heads (hint: `pbinom()`). This is NOT 0.5.
 - (c) I get at most 25 heads.
 - (d) I get at least 60 heads.

2. I take the QM5 every day to work from Manhattan to Queens. Traffic on the Queensboro Bridge means that I am sometimes late to work. The probability that I am late on any given day is 10%, and the traffic each day is independent. Assuming I work 300 days a year, how many days do I expect to be late on? What is the variance of the number of days I am late in a year? What is the probability that I will be late no more than 40 days in a year?

Hint: “being late on a day” is a Bernoulli variable with “success” probability 0.1. This question is about independent Bernoulli variables and their sums.

3. Here is some information about the monthly returns of two assets.

| Asset | Expected return | SD of return |
|-------|-----------------|--------------|
| X | 0.1 | 0.1 |
| Y | 0.05 | 0.05 |

The covariance between the assets’ returns is 0.003.

The standard deviation is the square root of the variance, so you can square it to get the variance.

- (a) What is the expected dollar payoff on a portfolio where I invest \$1 in each asset? What is the variance and standard deviation of that payoff?
- (b) What is the expected dollar payoff on a portfolio where I invest \$10 in X and \$5 in Y? What is the variance and standard deviation of that payoff?

Please do not use R for this question.

The way to answer these questions is to use the formulas. If X and Y are random variables and a and b are constants, then:

$$E(X + Y) = E(X) + E(Y)$$

$$E(X + a) = E(X) + a$$

$$E(bX + a) = bE(X) + a$$

$$V(X + a) = V(X)$$

$$V(a + bX) = V(bX) = b^2V(X)$$

$$Cov(aX, bY) = abCov(X, Y)$$

$$V(X + Y) = V(X) + V(Y) + 2Cov(X, Y)$$

For example, if I invest in asset X, I don't know what I'm going to make: my return is a random variable. I know I expect to make 10% (expected value of return is 10%), but I can make more than 10% or less than 10%.

Suppose I invest \$20 in asset X. I make R_X on that investment, but I don't know yet what R_X is going to be.

R_X will be a number like 0.09 (9%). If I invested \$1, I would get back $\$(1 + R_X)$. If I invest \$20, I get back 20 times that, or $\$20(1 + R_X)$. This is the random variable whose expected value and variance I want to find using the rules above.

What is my expected dollar payoff? It is the expected value of $20(1 + R_X)$, or

$$E(20(1 + R_X)) = E(20 + 20R_X) = 20 + E(20R_X) = 20 + 20E(R_X) = 20 + 20 * 0.1 = 22$$

This makes sense: I invest \$20, I expect to make back \$22 (an expected return of 10%).

What is the variance of my dollar payoff?

It is

$$V(20(1 + R_X)) = V(20 + 20R_X) = V(20R_X) = 20^2V(R_X) = 400 * (0.1^2) = 400 * 0.01 = 4$$

What about if I invested \$20 in X and \$30 in Y? Well, then my dollar payoff would be

$$20(1 + R_X) + 30(1 + R_Y)$$

And the expected value would be

$$E(20(1 + R_X) + 30(1 + R_Y)) = E(20 + 20R_X + 30 + 30R_Y) = 20 + 20ER_X + 30 + 30ER_Y = 20 + 20(0.1) + 30 + 30(0.1) = 50 + 5 = 55$$

And its variance would be

$$V(20(1 + R_X) + 30(1 + R_Y)) = V(20 + 20R_X + 30 + 30R_Y) = V(20R_X + 30R_Y)$$

(since numbers don't matter) And this is (using the rule for $V(X + Y)$)

$$V(20R_X + 30R_Y) = V(20R_X) + V(30R_Y) + 2Cov(20R_X, 30R_Y)$$

You can use the rules to say what each of these terms becomes. For example, the rule for $V(bX)$ says that the first term becomes $20^2V(R_X) = 400 * 0.01 = 4$.

Submission

You can report your answers in any form you like. I would prefer if you put them in a simple text file. For Q 1 and Q2, you could write down what command you input into R, what R responded, and how you used that response to answer the question. For Q3 you will have to write out your answer. Alternatively, you can write down your answers by hand and scan the paper into a PDF and upload that.