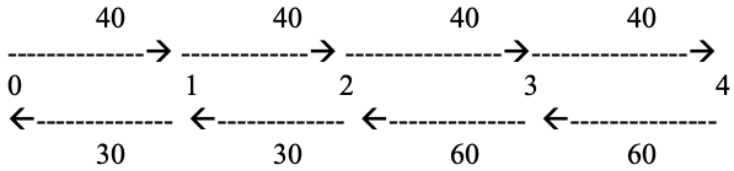
1. Customers arrive at a single-service facility at a Poisson rate of 40 per hour. When two or fewer customers are present, a single attendant operates the facility, and the service time for each customer is exponentially distributed with a mean value of two minutes. However, when there are three or more customers at the facility, the attendant is joined by an assistant and, working together, they reduce the mean service time to one minute. Assuming a system capacity of four customers,
   (a) what proportion of time are both servers free?
   (b) Each man is to receive a salary proportional to the amount of time he is actually at work servicing customers, the rate being the same for both. If together they earn $100 per day, how should this money be split?

(a) The system has given states whose transition diagram is



Hence, the balance equations are

$40P_0=30P_1$
$70P_1=40P_0+30P_2$
$70P_2=40P_1+60P_3$
$100P_3=40P_2+60P_4$
$60P_4=40P_3$

Solution of these gives $P_1=4/3P_0$, $P_2=16/9P_0$, $P_3=32/27P_0$, $P_4=64/81P_0$. The condition $\sum_0^4 P_i = 1$ implies $P_0=81/493$. Clearly, P_0=proportion of time both servers are free.

(b) The original attendant works a proportion $(1-P_0)$ of the time, and the second attendant works a proportion P_3+P_4 of the time. So if the first attendant receives $ x, the second should receive only $ $[P_3+P_4)/(1-P_0)]x$. So the first attendant receives $ 70.72 and the second attendant receives $ 29.28.

2. People's software company has just set up a call center to provide technical assistance on its new software package. Two technical representatives are talking the calls, where the time required by either representative to answer a customer's question has an exponential distribution with a mean of 8 minutes. Calls are arriving according to a Poisson process at a mean rate of 10 per hour. Determine L, Lq, W and Wq.

$\lambda=10$, $\mu=7.5$ $C=2$,
$L=2.4$, $Lq=L-Ls=2.4-\lambda/\mu=2.4-10/7.5=1.067$, $W=L/\lambda=0.24$, $Wq=Lq/\lambda=0.1067$