1. The manager of a market can hire either Mary or Alice. Mary, who gives service at an exponential rate of 20 customers per hour, can be hired at a rate $3 per hour. Alice, who gives service at an exponential rate f 30 customers per hour, can be hired at a rate of $C per hour. The manager estimates that, on the average, each customer's time is worth $1 per hour and should be accounted for in the model. If customers arrive at a Poisson rate of 10 per hour, then
   (a) What is the average cost per hour if Mary is hired? If Alice is hired?
   (b) Find C if the average cost per hour is the same for Mary and Alice.

Solution:

Let $C_M$=Mary's average cost/hour and $C_A$=Alice's average cost/hour .

Then $C_M$=$3+$1*(Average number of customers in the system when Mary works)

And $C_A$=$C+$1*(Average number of customers in the system when Alice works)

The arrival stream has parameter $\lambda=10$, and there are two service parameter-one for Mary and one for Alice:

$\mu_M = 20 \quad \mu_A = 30$

Set $L_M$= Average number of customers in the system when Mary works

$L_A$= Average number of customers in the system when Alice works

Then using Equation $L_M=10/(20-10)=1$

$L_A=10/(30-10)=1/2$

So $C_M$=$4 /hour

$C_A$=$C+1/2 /hour

(b) We can restate the problem this way. If $C_M=C_A$ then C=$3.5 /hour i.e., $3.5 /Hour is the most the employer should be willing to pay Alice to work. At a higher wage his average cost is lower with Mary working.

2. A facility produces items according to a Poisson process with rate $\lambda$.However, it has shelf space for only k items and so it shuts down production whenever k items are present. Customers arrive at the facility according to a Poisson process with rate $\mu$. Each customer wants one item and will immediately depart either with the item or empty handed if there is no item available.

(a) Find the proportion of customers that go away empty handed.
(b) Find the average time that an item is on the shelf
(c) Find the average number of items on the shelf.

This model is mathematically equivalent to the M/M/1 queue with finite capacity k. The produced items constitute the arrivals to the queue, and the arriving customers constitute the services. That is, if we take the state of the system to be the number of items presently available then we just have the model of Section 8.3.2.

(a) The proportion of customers that go away empty-handed is equal to $P_0$, the proportion of time there are no items on the shelves. Form Section 8.3.2,

$$P_0 = \frac{1-\lambda/\mu}{1-(\lambda/\mu)^{k+1}}$$

(b) $W = \dfrac{L}{\lambda(1-P_k)}$ Where L is given by equation 8.12

$$L = \frac{\lambda[1+k(\lambda/\mu)^{k+1}-(k+1)(\lambda/\mu)^k}{(\mu-\lambda)(1-(\lambda/\mu)^{k+1})}$$

$$P_k = \frac{(\lambda/\mu)^k(1-\lambda/\mu)}{1-(\lambda/\mu)^{k+1}}$$

(c)The average number of items in stock is L.