1. Machines in a factory break down at an exponential rate of six per hour. There is a single repairman who fixes machines at an exponential rate of eight per hour. The cost incurred in lost production when machines are out of services is \$10 per hour per machine. What is the average cost rate incurred due to failed machines?

2. Consider a single–server queue with Poisson arrivals and exponential service times having the following variation: Whenever a service is completed a departure occurs only with probability α. With probability $1-\alpha$ the customer, instead of leaving, joins the end of the queue. Note that a customer may be serviced more than once.

(a) Set up the balance equations and solve for the steady-state probabilities, stating conditions for it to exit.

(b) Find the expected waiting time of a customer from the time he arrives until he enters service for the first time.

(c) What is the probability that a customer enters service exactly *n* times, *n*=1, 2,…?

(d) What is the expected amount of time that a customer spends in service (Which does not include the time he spends waiting in line)?

3. Customers arrive at a two-server system according to a Poisson process having rate $\lambda=5$. An arrival finding server 1 free will begin service with that server. An arrival finding server 1 busy and server 2 free will enter service with server 2. An arrival finding both servers busy goes away. Once a customer is served by either server, he departs the system. The service times at server *i* are exponential with rates μ_i, where $\mu_1=4$, and $\mu_2=2$.

(a) What is the average time an entering customer spends in the system?

(b) What proportion of time is server 2 busy?

4. Customers arrive at a two-server system at a Poisson rate λ. An arrival finding the system empty is equally likely to enter service with either server. An arrival finding one customer in the system will enter service with the idle server. An arrival finding two others in the system will wait in line for the first free server. An arrival finding three in the system will not enter. All service times are exponential with rate μ, and once a customer is served (by either server), he departs the system.

(a) Define the states.

(b) Find the long-run probabilities.

(c) Suppose a customer arrives and finds two others in the system. What is the expected times he spends in the system?

(d) What proportion of customers enter the system?

(e) What is the average time an entering customer spends in the system?

5.

There are two types of customers. Type 1 and 2 customers arrive in accordance with independent Poisson Process with respective λ_1 and λ_2. There are two servers. A type 1 arrival will enter service with server 1 if that server is free; if server 1 is busy and server 2 is free, then the type 1 arrival will enter service with server2. If both servers are busy, then the type 1 arrival will go away. A type 2 customer can only be served by server 2; if server 2 is free when a type 2 customer arrives, then the customer enters service with the server. If server 2 is busy when a type 2 arrives, then that customer goes away. Once a customer is served by either server, he departs the system. Service times at server i are exponential with rate μ_i, $i = 1,2$.

Suppose we want to find the average number of customers in the system.

(a) Define states.

(b) Give the balance equations. Do not attempt to solve them.

In terms of the long-run probabilities, what is

(c) The average number of customers in the system?

(d) The average time a customer spends in the system?