1. This problem can be modeled by an M/M/1 queue in which $\lambda = 6, \mu = 8$. The average cost rate will be

$10 per hour per machine * average number of broken machines

The average number of broken machines is just L, which can be computed from equation

$$L = \frac{\lambda}{\mu - \lambda} = \frac{6}{2} = 3$$

Hence, the average cost rate = $30/hour.

2

(a)

$$\lambda P_0 = \alpha\mu P_1$$
$$(\lambda + \alpha\mu) P_n = \lambda P_{n-1} + \alpha\mu P_{n+1} \quad \text{n>=1}$$

These are exactly the same equations as in the M/M/1 with $\alpha\mu$ replacing μ.

Hence,

$$P_n = [\frac{\lambda}{\alpha\mu}]^n [1 - \frac{\lambda}{\alpha\mu}],\ n\text{>=0}$$

And we need the condition $\lambda < \alpha\mu$

(b) If T is the waiting time until the customer first enters service, then by conditioning on the number present when he arrives yields

$$\text{E}[T] = \sum_n \text{E}[T \mid n \text{ present}] * P_n$$
$$= \sum_n \frac{n}{\mu} P_n = \frac{L}{\mu}$$

Since $L = \dfrac{\lambda}{\alpha\mu - \lambda}$

So $\text{E}[T] = \dfrac{\lambda}{\mu(\alpha\mu - \lambda)}$

(c)P{enters service exactly n times}$= (1-\alpha)^{n-1}\alpha$

(d)This is expected number of services and mean service time $= \dfrac{1}{\alpha\mu}$

(e) What is the distribution of the total amount of time that a customer spends in service? The distribution is easily seen to be memoryless. Hence, it is exponential with rate $\alpha\mu$.

3. The state space can be taken to consist of states (0, 0), (0, 1), (1, 0), (1, 1), where the ith component of the state refers to the number of customers at server i, i=1, 2. The balance equations are

$5P_{0,0}=2P_{0,1}+4P_{1,0}$
$7P_{0,1}=4P_{1,1}$
$9P_{1,0}=5P_{0,0}+2P_{1,1}$
$6P_{1,1}=5P_{0,1}+5P_{1,0}$
$1=P_{0,0}+P_{0,1}+P_{1,0}+P_{1,1}$

Solving these equations gives $P_{0,0}=128/513$, $P_{0,1}=100/513$, $P_{1,0}=110/513$, $P_{1,1}=175/513$

(a) $W=\dfrac{L}{\lambda_a}=[1(P_{0,1}+P_{1,0})+2P_{1,1}]/[\lambda(1-P_{1,1})]=\dfrac{56}{169}$

(b) $P_{0,1}+P_{1,1}=275/513$

4

(a) The states are 0, 1, 2, 3 where the state is *i* when there are *i* customers in the system.

(b) The balance equations are

$$\lambda P_0 = \mu P_1$$
$$(\lambda+\mu)P_1 = \lambda P_0 + 2\mu P_2$$
$$(\lambda+2\mu)P_2 = \lambda P_1 + 2\mu P_3$$
$$2\mu P_3 = \lambda P_2$$
$$1 = \sum_{j=0}^{3} P_j$$

$P_1=(\lambda/\mu)P_0$, $P_2=(\lambda^2/2\mu^2)P_0$, $P_3=(\lambda^3/4\mu^3)P_0$, $P_0=[1+\lambda/\mu+(\lambda^2/2\mu^2)+(\lambda^3/4\mu^3)]^{-1}$

(c) E[Time | find two others in the system]$=(1/2\mu)+(1/\mu)$

(d) $1-P_3=\dfrac{4\mu^3+4\lambda\mu^2+2\mu\lambda^2}{4\mu^3+4\mu^2\lambda+2\mu\lambda^2+\lambda^3}$

(e) $W=\dfrac{L}{\lambda(1-P_3)}=\dfrac{4\mu^2+4\lambda\mu+3\lambda^2}{4\mu^3+4\mu^2\lambda+2\mu\lambda^2}$

$L=0*P_0+1*P_1+2*P_2+3*P_3$

5.

(a) The states are (0, 0), (1, 0), (0, 1) and (1, 1), where (0, 0) means that the system is empty, (1, 0) means that there is one customer with server 1 and none with server 2, and so on.

(b)

$$(\lambda_1+\lambda_2)P_{00} = \mu_1 P_{10} + \mu_2 P_{01}$$
$$(\lambda_1+\lambda_2+\mu_1)P_{10} = \lambda_1 P_{00} + \mu_2 P_{11}$$
$$(\lambda_1+\mu_2)P_{01} = \lambda_2 P_{00} + \mu_1 P_{11}$$
$$(\mu_1+\mu_2)P_{11} = \lambda_1 P_{01} + (\lambda_1+\lambda_2)P_{10}$$
$$P_{00}+P_{01}+P_{10}+P_{11}=1$$

(b) $L=1*(P_{01}+P_{10})+2*P_{11}$

(c) $W=\dfrac{L}{\lambda_a}=\dfrac{L}{\lambda_1(1-P_{11})+\lambda_2(P_{00}+P_{10})}$