Matlab Assignment for 251

General Information:

- The Matlab website is full of useful information if you want to learn more Matlab Support
- The University Software Portal has information on how to download Matlab or if preferred how to work with the online version of Matlab.

Part 1

Parametrizing Curves:

Viviani's curve is defined as the intersection of the sphere $x^2 + y^2 + z^2 = 4$ with the cylinder $(x-1)^2 + y^2 = 1$. Observe that

$$\begin{cases} x = 1 + \cos t \\ y = \sin t \\ z = \pm 2 \sin\left(\frac{t}{2}\right) \end{cases}$$

gives a parametrization of this curve [this uses the identity $\sin\left(\frac{t}{2}\right) = \pm\sqrt{\frac{1-\cos t}{2}}$].

To plot Viviani's curve **use the following code**

```
>> syms t; x=1 +cos(t); y=sin(t), z=2*sin(t/2);
>> fplot3(x,y,z, [0 pi])
```

Exercise 1. Save the image you get as a pdf or jpeg file.

The figure you obtained should not be terribly illuminating. We ca get rid of the grid that by default surrounds all three-dimensional graphics in MATLAB. We can dispense with the labels ad tick marks on the axes. It is also helpful to show the curve inside the sphere and the cylinder that are used to define it. We can show the arcs obtained by intersecting with the coordinate planes, as well as the projected circle on the xy plane.

```
>> hold on
>> fplot3(sym(0), 2*cos(t/2), 2*sin(t/2), [0 pi])
>> fplot3(2*cos(t/2), sym(0), 2*sin(t/2), [0 pi])
>> fplot3(2*cos(t/2), 2*sin(t/2), sym(0), [0 pi])
>> fplot3(1+cos(t), sin(t), sym(0), [0 pi])
>> fplot3(sym(0), sym(0), t, [0 2])
>> fplot3(sym(0), t, sym(0), [0 2])
>> fplot3(t, sym(0), sym(0), [0 2])
>> title(' '); xlabel(' '); ylabel(' '); zlabel(' ');
>> grid off; axis off;
view ([10,3,1])
```

Exercise 2. Save the image you get as a pdf or jpeg file.

Exercise 3. Identify which fplot3 commands correspond to

- **The x-axis, y-axis, z-axis**
- **Circle arc on the xy plane, xz plane, yz plane**
- **Circle $(x-1)^2+y^2=1$ on the xy plane**
- **You can enter comments on Matlab directly by using the % symbol.**

Exercise 4. Now plot the curve which is obtained as the intersection of the paraboloid $z=x^2+y^2$ with the cylinder $(x-1)^2+y^2=1$. Use the interval [0 2*pi] instead of [0 pi]. Hint: notice that this is the same cylinder as before so x and y are given by the same functions of t. Only z changes.

Graphing Functions:

fsurf and fmesh are the basic MATLAB commands for graphing a function of two variables. For example, suppose we want to graph the function

$$f(x,y)=y\left(1-\frac{1}{(x^2+y^2)}\right)$$

Exercise 5. What is the domain of this function?

To plot it write

```
>> flowfunction= @(x,y) y*(1-1/(x^2+y^2));
>> figure; fsurf(flowfunction, [-2 2 -2 2])
```

You will receive a warning because the function is not defined everywhere on the xy plane but that is ok.'

Exercise 6. Save the image you get as a pdf or jpeg file.

To graph level curves we can use:

```
>> list1= -2:0.5:2; figure; fcontour(flowfunction, [-2 2 -2 2], 'LevelList', list1);
```

Exercise 7. Save the image you get as a pdf or jpeg file.

To shade the regions between the level curves selected use

```
>> figure; fcontour(flowfunction, [-2 2 -2 2], 'LevelList', list1, 'Fill', 'on')
```

Exercise 8. Save the image you get as a pdf or jpeg file.

Exercise 9. Plot the function $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$**, and define list1=[-0.2, -0.1], list2=[0.1, 0.2, 1, 4, 10], list3=[0] Compare the level curves of** f **for these different values. Save the images you get. Use the grid [-1 1 -1 1]**

Part 2

Surfaces and Vector Fields:

```
>> syms x y z; h=z^2 -x^2-y^2;
>> fimplicit3(h-1, [-1.1 1.1 -1.1 1.1 -2 2]); axis equal
```

Exercise 10. Save the image you get as a pdf or jpeg file.

With the command quiver3 we can visualize gradient vectors. Write

```
>> [X,Y]=meshgrid(-1:0.5:1., -1:0.5:1.);
>> Z=sqrt(1+X.^2+Y.^2);
>> [U,V,W]=surfnorm(X,Y,Z);
>> hold on; quiver3(X,Y,Z,U,V,W,2)
>> Z1= -sqrt(1+X.^2+Y.^2);
>> [U,V,W]= surfnorm(X,Y,Z1);
>> quiver3(X,Y,Z1,U,V,W,2)
```

Exercise 11. Save the image you get as a pdf or jpeg file.

Exercise 12. Plot the ellipsoid $f(x, y, z) = x^2 + 2y^2 + 3z^2 = 1$ **together with the tangent plane at the point** $(1, 0, 0)$

Parametrizing Surfaces:

A parametrization of the sphere $x^2 + y^2 + z^2 = 1$ in spherical coordinates θ, ϕ:

$$\begin{cases} x = \cos u \sin v \\ y = \sin u \sin v \\ z = \cos v \end{cases}$$

Here u is playing the role of θ while v is playing the role of ϕ.

To plot the sphere use

```
>> syms u v real; sphere= [cos(u)*sin(v), sin(u)*sin(v), cos(v)];
>> fsurf(sphere(1), sphere(2), sphere(3), [0 2*pi 0 pi])
>> view([1,1,1]); title(' '); xlabel(' '); ylabel(' '); zlabel(' ');
>> grid off; axis equal; axis off;
```

Exercise 13. Save the image you get as a pdf or jpeg file.

Exercise 14. Parametrize the monkey saddle $z = x^3 - 3xy^2$ by using $u = x$, $v = y$ as the parameters. Notice that here it makes more sense to use both negative and positive values of u, v

Exercise 15. Save the image you get as a pdf or jpeg file.