

# Question

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I'm writing a dissertation about bootstrap methods and the main book I'm using is "Efron, B., & Tibshirani, R.J. (1994). An Introduction to the Bootstrap(1st ed.). Chapman and Hall/CRC".

Now I need to deal with the bootstrap-t confidence intervals but I got many doubts about it. I'm not a native speaker but I'll try to be as clear as possible.

We're in a situation where  $(x_1, \dots, x_n)$  is a fixed data set from an unknown distribution  $F$ ,  $\theta$  is a parameter, not necessary the mean, and  $\hat{\theta} = s(X_1, \dots, X_n)$  is an estimator. We denote with  $\mathbf{x}_1^*, \dots, \mathbf{x}_B^*$   $B$  bootstrap samples and respectively with  $\hat{\theta}^*(1), \dots, \hat{\theta}^*(B)$  the bootstrap replications. Following pages 160-161 of Efron's book, it uses the following estimator to find bootstrap-t intervals:  $Z_b^* = \frac{\hat{\theta}^*(b) - \hat{\theta}}{\hat{s}e^*(b)}$ , where  $\hat{s}e^*(b)$  is the standard error of  $\hat{\theta}^*(b)$ . Now I got some questions to ask:

1. Why he took exactly  $Z_b^*$  and not, for example, just  $\hat{\theta}^*(b) - \hat{\theta}$ ? In fact, we can't say that  $Z_b^*$  is normal or Student, we can't even say that it is close to a Student because we don't know the distribution  $F$  and we don't know what is  $\theta$ . So we could find  $\alpha$ th percentiles that Efron finds in the same way with  $\hat{\theta}^*(b) - \hat{\theta}$ .
2. Why the intervals we found are for  $\theta$ , when in  $Z_b^*$  the parameter  $\theta$  doesn't appear?
3. Why in  $Z_b^*$  we use  $\hat{s}e^*$  instead of  $\hat{s}e$ ?
4. the term  $\hat{\theta}$  in  $Z_b^*$  is for  $s(X_1, \dots, X_n)$ , so a random variable, or for  $s(x_1, \dots, x_n)$ , so a fixed value?
5. Which are the analogies with the t-Student?