Please read and signed before you begin the exam.

Pledge:

"As a student of the University of Miami, I commit myself to being an active member in the Academic Community of Trust By promoting the values of Honesty, Responsibility, and Integrity."

Moreover, I pledge, on my honor, that I have neither given nor received assistance on this Exam, (in particular I pledge that I will not communicate (this includes emailing) with anyone after the exam about the material on this exam until the professor gives me permission to do so).

Signed,

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Please write the following on your exam "I acknowledge that I have read and understand the pledge"

Then sign it.

1 Econometrics Final Exam (Eco 620)

Directions: Directions: This is a take-home exam. Students are NOT allowed to collaborate. There are 5 questions with various parts. Make sure you write your name as it appears on your ID so that you can receive the correct grade. The exam due date is **Wedneday May 6 at 9:30am** any exam return after this deadline will not be accepted. Student must signed the Pledge and upload it as the first page on the answer sheet. Good luck!

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Q1.

Assume that we have the following structural model,

$$y_{1t} = \alpha_{11}y_{2t} + \alpha_{12}y_{3t} + \alpha_{13}y_{4t} + \beta_{11}x_{1t} + \beta_{12}x_{2t} + \varepsilon_{1t}$$

a. What is the identification condition - how many exogenous variables must be excluded for the above equation to be identified (assuming that the equation is exactly identified)?

b. What is/are the first stage regressions? (assume the excluded exogenous are x_{4t} etc. and proceed).

c. What is/are the second stage regressions, (please explain and demonstrate why 2SLS estimates are consistent).

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Q2.

The linear regression model in vector form is given as,

$$\mathbf{y}_1 = Y_1\alpha_1 + X_1\beta + \varepsilon_1 = Z_1\delta_1 + \varepsilon_1$$

where $\mathbf{y}_1$ is a $T \times 1, Y_1$ is a $T \times g_1$, α_1 is a $g_1 \times 1$, X_1 is a $T \times k_1$, β is a $k \times 1$ and ε_1 is a $T \times 1$). Let X_2

be a $T \times k_2$, matrix that represents the excluded exogenous variables which appear in the other equations in the system.

Let $X = [X_1,\ X_2]$, $Z_1 = [Y_1, X_1]$ and $\delta_1^{'} = (\alpha_1^{'}, \beta_1^{'})$. The 2SLS estimator is given as, $\widehat{\delta}_{1,2sls} = (Z_1^{'} P_X Z_1)^{-1} Z_1^{'} P_X \mathbf{y}_1$ where $P_X = X(X^{'}X)^{-1}X^{'}$.

a. Suppose the model is just identified show that the above estimator reduces to $\widehat{\delta}_{1,2sls} = (X^{'}Z_1)^{-1}X^{'}\mathbf{y}_1$ (provide any assumption that is required to show this).

b. Given the information in part "a", let W be a set of instruments and suppose $W = X$ what can we conclude about 2SLS and the simple IV estimator when the model is just identified?

c. Explain how to conduct a Hausman's specification test. Discuss the properties of the estimators, H_0, $H_{1,}$ the level of significance, the test statistics, the distribution of the test statistics and the conclusion.

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Q3.

Assume the following model,

$$y_t = \beta x_t + \varepsilon_t, \text{ for } t = 1, ..., T$$

assume that x_t is endogenous and you find an instrument w_t. The first stage regression is,

$$x_t = \gamma w_t + u_t, \text{ for } t = 1, ..., T$$

Assume that $\varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$ and $u_t \overset{iid}{\sim} N(0, \sigma_u^2)$. Assume that the instrumental variable is irrelevant (using an F-test we cannot reject the null that $\gamma = 0$).

a. Derive $\widehat{\beta}_{iv}$ (**note you must show the derivation**)

b. Is $\widehat{\beta}_{iv}$ consistent, show it?

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Q4.

For panel data. In vector notation: $y = \alpha i_{NT} + X\beta + \varepsilon$ and $\varepsilon = Z_\mu \mu + v$

$\Rightarrow$

$$y = \alpha i_{NT} + X\beta + Z_\mu \mu + v$$

where $Z = [i_{NT}, X]$, $\delta^{'} = [\alpha^{'}, \beta]$. Note: y is $NT \times 1, X$ is $NT \times k$; α is scalar, β is $k \times 1$ and ε is $NT \times 1$. Let $Z_\mu = I_N \otimes i_T$ and let $P = Z_\mu (Z_\mu^{'} Z_\mu)^{-1} Z_\mu^{'}$ be the projection matrix on Z_μ. Let $Q = I_{NT} - P$ where P and Q are idempotent matrices.

a. Suppose the true model is the equation above, what are the properties of pooled OLS estimate, please explain?

b. Derive the slope estimate for the true model and call it $\widehat{\beta}_{LSDV}$ equation 1

c. Pre-multiply the equation above by Q and derive $\widetilde{\beta}_w$ (the within estimator)

Asumme that model is given in scalar from such that,

$$y_{it} = \alpha + \beta x_{it} + \mu_i + v_{it}.$$

The estimator for the fixed effects or the unobserved heterogeneiety (μ_i) is given as,

$$\widehat{\mu} = (\overline{y}_{i.} - \overline{y}_{..}) - \widetilde{\beta}_w(\overline{x}_{i.} - \overline{x}_{..}), \text{ where } \overline{y}_{i.} = \frac{\sum_{t=1}^{T} y_{it}}{T}, \overline{y}_{..} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} y_{it}}{NT}$$

$$\overline{x}_{i.} = \frac{\sum_{t=1}^{T} x_{it}}{T} \text{ and } \overline{x}_{..} = \frac{\sum_{i=1}^{N}\sum_{t=1}^{T} x_{it}}{NT}.$$

$$\text{use the condition that } \sum_{i=1}^{N} \mu_i = 0$$

d. Is the estimator $(\widehat{\mu})$ unbiased, show it ?

e. Is the estimator $(\widehat{\mu})$ consistent, show it?

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Q5.

Consider the simple logit regression model:

$$y_t = F(\beta x_t) + \varepsilon_t$$

for $t = 1, 2$ (two time periods). Let $\beta = 1$ and x_t takes on fixed values such that $x_1 = 1$ and $x_2 = 2$. The $\varepsilon_t^{'}$s are independent with mean zero. Assume that the CDF $(F(\beta x_t))$ follows a logit distribution.

a. Derive the sampling distribution of the least squares estimator of β and call it $\widehat{\beta}_{ols}$ assuming a linear probability model (LPM) when the true model is a logit model.

b. What is the $E(\widehat{\beta}_{ols})$, is OLS unbiased/biased show it?