Study Area 2: Applications of Calculus Task (15% of unit mark)

Specifications

Task Type	This assessment is a group application and writing task ▪ Complete in a group of 2 or 3 ▪ 3 Problem Based Tasks
Scope of Task	This assessment covers learning outcomes contained in Study Area 2.
Assessment Conditions	Students work in groups (maximum of 3 members) to complete the assignment, collaborating via digital technology. Groups will be observed by the teacher in a 55-minute Zoom session in which students will work together in breakout rooms. All students must access the Application Task on Moodle and submit onto Moodle their own Answer Summary containing the agreed answers for their group. The peer review for the task must be completed on Moodle
Duration	One (1) week

(THIS DOCUMENT IS FOR WORKING ONLY)

[THIS SHOULD NOT BE UPLOADED]

MARKS

TOTAL MARKS	PERCENTAGE	GRADE
/20	%	

Information for students

- Working in groups (maximum of 3 members), students will be allowed **one week** to collaborate in solving the problems in the task and communicating their findings.

- Each group will need to establish a viable method of communication. Some Zoom time will be provided.

- Students should decide how they will approach the task in their groups. All group members must be prepared to make a fair and reasonable contribution to the group effort.

- Every member of a group must access the assignment from LMS and work individually on the questions until they are ready to share their ideas with the rest of the group. The answers recorded by each student will be the ones that the group members have accepted as the best. Each student will complete an Answer Summary by typing the group's consensus answers into their own Summary Sheet.

- An Answer Summary from every student must be submitted onto Moodle in accordance with the set deadline. Instructions for doing this will be made available.

- Students will be assessed on the criteria set out in the assessment rubric.

- Students may refer to their notes, textbooks and online resources, but discussion of the assignment with others should be restricted to members of their group.

- At any time, students may seek clarification from a teacher regarding the wording of information or questions, but teachers may not help with mathematical calculations or with explanations of mathematical terminology.

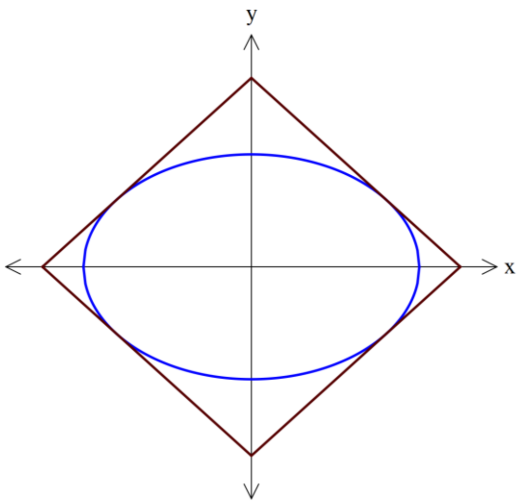
Assessment criteria

- The Group Application Task will be marked using the Group Task rubric. It accounts for 15% of the overall mark for the subject.

- Students are expected to complete a peer review for the task on LMS by the deadline set by the teacher. Failure to do so on time may affect the collaboration score.

Task 1

An outdoor sports complex is located in a *field* in the shape of an ellipse given by the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$. The *field* is surrounded by a *fence* in the shape of a parallelogram as shown in the diagram. The *fence* touches the edge of the *field* in four places.



a) Find $\frac{dy}{dx}$ for the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$.

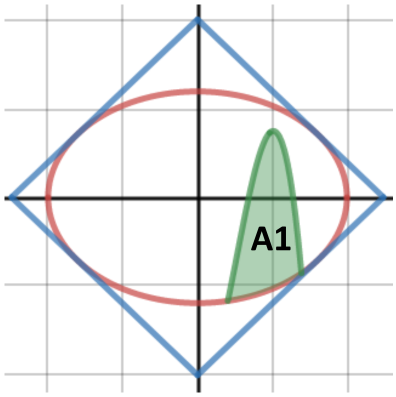
b) The sides of the fence are made up of four straight lines. Find these equations given that the fence touches the edge of the field at $x = \pm 4$.

c) Explain why the sides of the fence must be a rhombus.

d) Find the exact area of the region enclosed by the four fences.

Task 2

Part of the *field* will be used to for the sport *javelin* throw. The *javelin* area is defined by **A1.** The area is enclosed by the curve $f(x) = -x^3 + 4x^2 - x - 5$ and the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$. The area **A1** will be filled with grass for the *javelin* throw.



a) Determine, the exact x-value within the *field* where the tangent line of $f(x) = -x^3 + 4x^2 - x - 5$ is parallel to the line $g(x) = 2x - 7$.

The *javelin* area needs to be filled with grass.

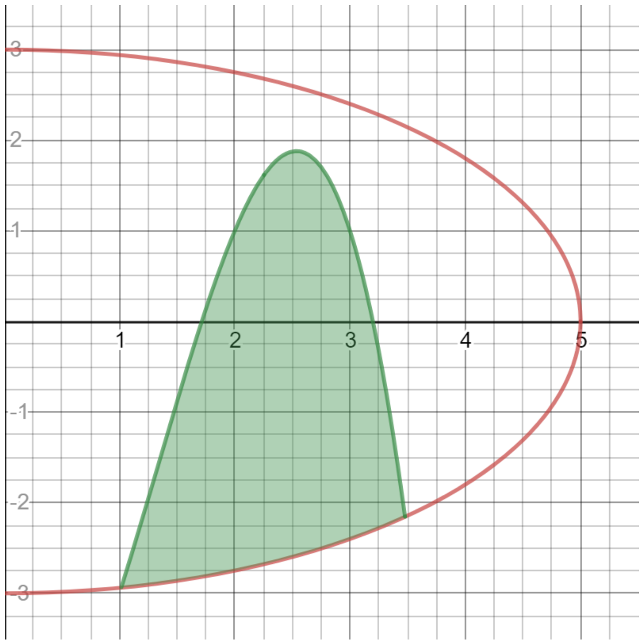
Pre-cut grass can only be laid if the curve $f(x) = -x^3 + 4x^2 - x - 5$ is concave up within the *field.* If the curve is concave down within the *field,* seeds will need to be planted.

b) Find the inflection point(s) and the intervals of concavity up and down of the *javelin* area within the domain $x \in [0,5]$.

c) Describe how the *javelin* area can be filled with grass within the domain $x \in [0,5]$.

d) Express the equation of the ellipse as $y = h(x)$.

e) Using your calculator or otherwise, find the coordinates of the intersection points of the graphs $f(x)$ with the edges of the *field* correct to 1 decimal place. Label these on the given diagram.



f) Write down the integral expression for the area to be filled with grass to cover **A1**.
Use values from part e).

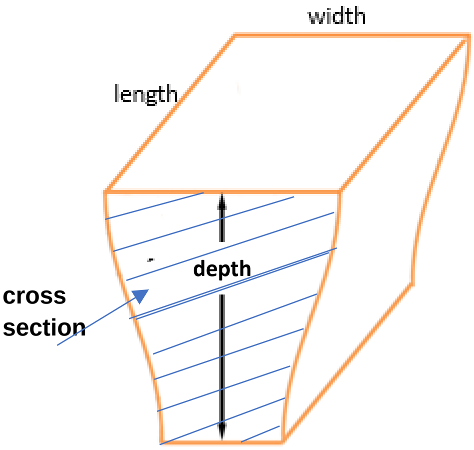
g) Hence, using your calculator determine the area **A1** correct to the nearest unit.

Task 3

Sand is required for the *sandpit* that is used for the *long jump* event. The *sandpit* is located within the field.

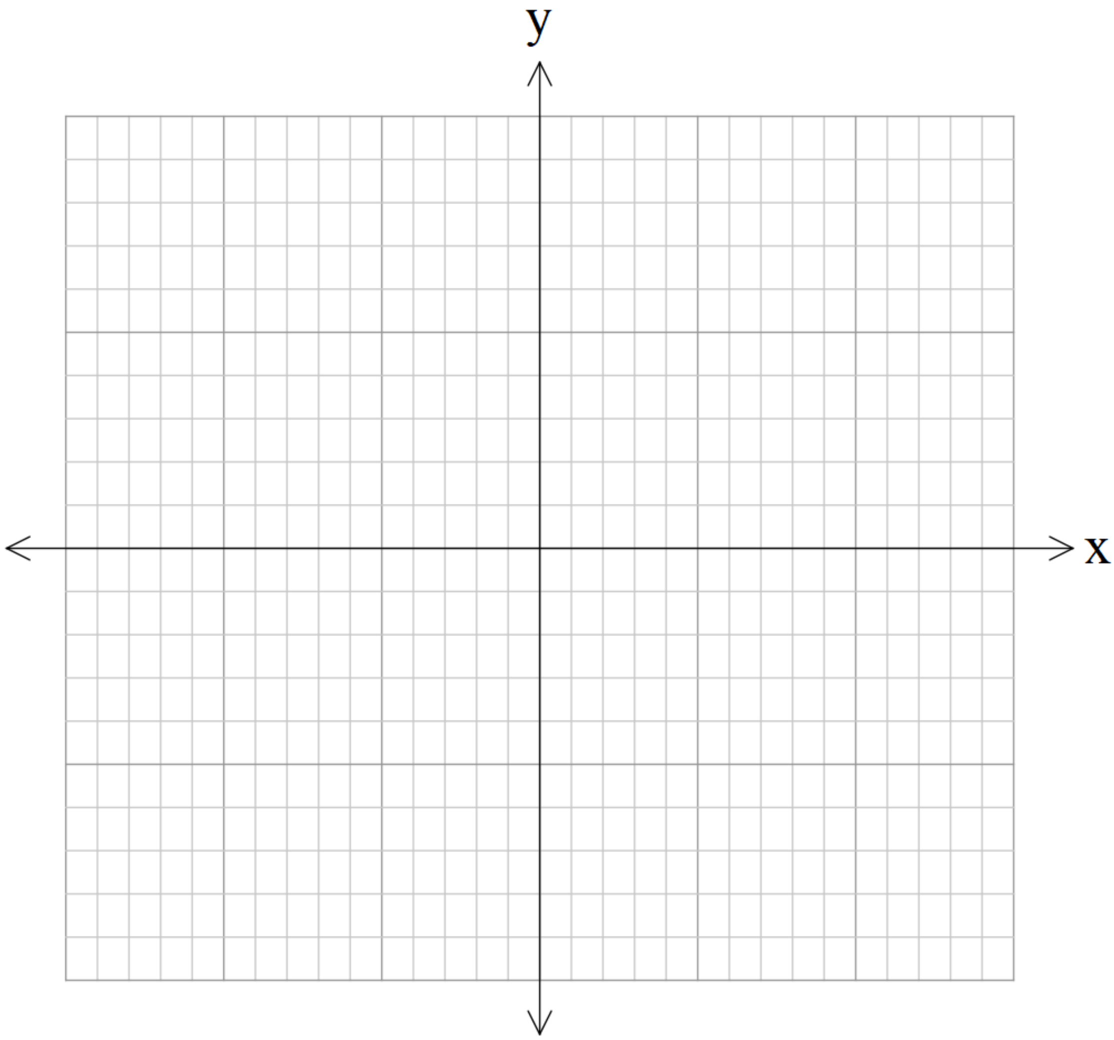
The surface area of the sandpit is rectangular in shape where the length is 10 metres and the width is $2\left(\frac{b}{10}\pi + 1\right)$ metres.

The shape of the cross section of the *sandpit* (shaded below) is enclosed by the equations $g(x) = \frac{a}{\pi}\cos^{-1}\left(\frac{b}{10}\pi \pm x\right)$ and the depth by $y = 0$ and $y = a$.



a) Let $a = 1$ and $b = 5$. Find the domain and range of the two functions given by the equations of *g(x).*

Hence, sketch the graphs of the function $g(x) = \frac{a}{\pi}\cos^{-1}\left(\frac{b}{10}\pi \pm x\right)$ using $a = 1\ and\ b = 5$. Label the depth of the *sandpit* and determine the width.



b) Do the values for a and b represent a realistic looking *sandpit*? Explain.

In your group, select the last two digits of the student number of Member 1. Use the table below to find the a and b values that will form the *sandpit.* Repeat this process with each team member.

Second last digit	0	1	2	3	4	5	6	7	8	9
a	1	1	2	3	4	5	6	7	8	9
Last digit	0	1	2	3	4	5	6	7	8	9
b	5	6	7	8	9	10	11	12	13	14

c) Using your calculator plot *g(x)* for your values. By trying these different *a* and *b* values, explain how the shape of the cross section of the *sandpit* changes.

d) Show that the area of the cross section of the *sandpit* is $A = \frac{b\pi h}{5} - \frac{2a}{\pi}\sin\left(\frac{\pi h}{a}\right)\text{m}^2$, where *h* represents the depth of the *sandpit* and $0 \le h \le a$.

e) Hence, given that $h = a$, show that the volume of the sand pit is $V = 2b\pi h\, m^3$.

f) Given that $a = 1,\ b = 5,$ evaluate the area of the cross section for your *sandpit* as well as the volume of the sandpit.

g) The *sandpit* is initially empty. Sand is poured into the pit at a rate of $5m^3/min$. The sand forms a right circular cone with the height equal to *twice* its radius. Find the rate of change in m/min of the radius of the cone when the volume of sand is $V = \frac{ab\pi}{3}\ m^3$.

(Volume of a right circular cone: $V = \frac{1}{3}\pi r^2 h$)

h) Sand is entering the *sandpit* at the same rate of $5m^3/min$. Using part e), find an expression for the rate of change in m/min at which the height of the sand in the *sandpit* is increasing.

MONASH UNIVERSITY FOUNDATION YEAR

MUF0102 ADVANCED MATHEMATICS UNIT 2: CALCULUS WITH APPLICATIONS

FORMULA SHEET

Circular (trigonometric) functions

$\frac{1}{\sin(x)} = \operatorname{cosec}(x)$	$\frac{1}{\cos(x)} = \sec(x)$
$\frac{1}{\tan(x)} = \cot(x)$	
$\sin^2(x) + \cos^2(x) = 1$	
$1 + \tan^2(x) = \sec^2(x)$	$\cot^2(x) + 1 = \operatorname{cosec}^2(x)$
$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y)$	$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$
$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$	$\cos(x-y) = \cos(x)\cos(y) + \sin(x)\sin(y)$
$\tan(x+y) = \frac{\tan(x) + \tan(y)}{1 - \tan(x)\tan(y)}$	$\tan(x-y) = \frac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$
$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$	
$\sin(2x) = 2\sin(x)\cos(x)$	$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$

Inverse Circular Functions

Function $y = f(x)$	Domain	Range	$x = g(y)$	$y = f'(x)$
$y = \sin^{-1} x$	[−1, 1]	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$	$x = \sin y$	$\frac{1}{\sqrt{1-x^2}}, -1 < x < 1$
$y = \cos^{-1} x$	[−1, 1]	$[0, \pi]$	$x = \cos y$	$\frac{-1}{\sqrt{1-x^2}}, -1 < x < 1$
$y = \tan^{-1} x$	$(-\infty, \infty)$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$x = \tan y$	$\frac{1}{1+x^2}$

CALCULUS

$\frac{d}{dx}(x^n) = nx^{n-1}$ where n is a rational number	$\int x^n\,dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1,$ where n is a rational number
$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}dx = \frac{1}{a}e^{ax} + c$
$\frac{d}{dx}(log_e x) = \frac{1}{x},$ where $x > 0$	$\int \frac{1}{x}dx = log_e\lvert x\rvert + c$, where $x > 0$
$\frac{d}{dx}(sin\,a\,x) = a\,cos\,a\,x$	$\int \sin ax\,dx = -\frac{1}{a}\cos ax + c$
$\frac{d}{dx}(cos\,a\,x) = -a\,sin\,a\,x$	$\int \cos ax\,dx = \frac{1}{a}\sin ax + c$
$\frac{d}{dx}(tan\,a\,x) = a\,sec^2\,a\,x$	$\int sec^2\,ax\,dx = \frac{1}{a}tan\,a\,x + c$
	$\int (ax+b)^n dx = \frac{1}{a(n+1)}(ax+b)^{n+1} + c, n \neq -1$
Integration by Parts	$\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$
Product rule	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$
Quotient rule	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$
Chain rule	$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
Acceleration	$a = \frac{\partial^2 x}{\partial t^2} = \frac{dv}{dt} = v\frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$
Arc length	$\int_{x_1}^{x_2}\sqrt{1+(f'(x))^2}dx$ or $\int_{t_1}^{t_2}\sqrt{(x'(t))^2+(y'(t))^2}dt$

Category	Excellent 4 marks	Good 3 marks	Satisfactory 2 marks	Needs Improvement 1 mark	Not Attempted 0 marks
Knowledge Recall, Technical Skills and Application	Thorough understanding of mathematical ideas and relationships. Effective application of skills.	Mostly thorough understanding of mathematical ideas and relationships. Mostly effective application of skills.	Adequate understanding of mathematical ideas and relationships. Some effective application of skills.	Basic understanding of mathematical ideas and relationships. Limited demonstration of application of skills.	Not demonstrated.
Communication of Mathematical Ideas	Effective use of reasoning and logic. Correct use of mathematical notation and terminology.	Mostly effective use of reasoning and logic. Mostly correct use of mathematical notation and terminology.	Some effective use of reasoning and logic. Some correct use of mathematical notation and terminology.	Limited demonstration of reasoning and logic. Mostly correct use of mathematical notation and terminology.	Not demonstrated.
Analysis and Use of Technology	Insightful application of mathematical processes to generate solutions and check for reasonableness. Effective use of technology skills to solve problems.	Proficient application of mathematical processes to generate solutions and check for reasonableness. Mostly effective use of technology skills to solve problems.	Variable application of mathematical processes to generate solutions and check for reasonableness. Some use of technology skills to solve problems.	Minimal application of mathematical processes to generate solutions and check for reasonableness. Limited use of technology skills to solve problems.	Not demonstrated.
Use of Language	Concise and accurate use of language which is appropriate for the purpose; some minor error do not detract from the overall fluency.	Mostly accurate use of language which is appropriate for the purpose; errors are noticeable but do not detract from the overall fluency.	Some accurate use of language which is generally appropriate for the purpose; occasional errors detract from the fluency.	Frequent use of language which may be inappropriate or inconsistent; persistent errors interfere with meaning.	Not demonstrated.
Collaboration	Effective teamwork skills, strong communication (in English) with other members.	Mostly effective teamwork skills, sound communication (in English) with other members.	Moderate teamwork skills, some communication (in English) with other members.	Basic teamwork skills, limited communication (in English) with other members.	Not demonstrated.

MARKS

TOTAL MARKS	GRADE	PERCENTAGE
/20		%