Problem C: Airplane weight and safe landing time

When a big jetliner lands on a runway, this produces a lot of turbulence in the air along the runway, which is potentially dangerous for the next airplane trying to land on the same runway some time later. Therefore, it used to be standard procedure to wait for two minutes for the air to calm down before the next airplane is allowed to land. However, with the introduction of the Airbus, even larger and heavier than the jetliners before it, this is not enough. Even as long as two minutes after an Airbus lands, there still is so much turbulence that the next airplane may get in trouble.

So those new Airbuses may be capable of carrying more freight per airplane, but the number of airplanes that can land at an airport per unit of time decreases, because they have to wait longer for the turbulence to disappear. Consequently, the capacity of the airport may actually decrease instead of increase with the introduction of larger airplanes...

In this problem you will investigate the consequences of this weight-dependent waiting time on the optimal weight of airplanes¶. Suppose that the new safety rule will be that airplanes have to wait for a period

$$\tau(w) = \mathrm{e}^{2w}, \tag{17}$$

where w is the total weight of the (previous) airplane in Gg = 1000 tonnes = 10^6 kg and $\tau(w)$ is the required waiting time in minutes.

(a). For what weight of an airplane does this new rule amount to the same waiting time as the former two-minutes rule?

We are interested in the amount of cargo (passengers or freight) that can be brought in through the airport per unit of time. To determine the actual net weight of the cargo, we need to take into account the empty weight w_0 of the Airbus, which is about 250 tonnes = 0.250 Gg.

(b). Argue that the net rate of cargo that can be brought in per hour by Airbuses landing on a single runway is given by

$$q(w) = 60\,(w - 0.25)\,\mathrm{e}^{-2w}. \tag{18}$$

What implicit assumption do you make about the diversity of airplanes landing on this runway to derive this equation?

(c). Find out what the optimal weight of an Airbus would be; what is the amount of cargo per Airbus such that the net rate of cargo per hour would be optimized‖?

In fact, the maximum (total) weight w_m that the Airbus can carry is 600 tonnes = 0.600 Gg. If it is heavier than that, the airplane cannot remain airborne**.

(d). What is the appropriate domain for the function $q(w)$ in (18)? Consider the optimal cargo load of Airbuses in view of this restriction.

New aircraft are being designed as we speak. Those will have a different empty weight w_0 and maximum weight w_m than the Airbus. These values would be given for a given aircraft, once it's been designed, but at the moment we don't know their exact values yet.

(e). Argue that in this case the net rate of cargo that can be brought in per hour on a single runway is given by

$$q(w) = 60\,(w - w_0)\,\mathrm{e}^{-2w}. \tag{19}$$

What assumption did you make about the validity of the rule (17) to derive this equation?

¶Of course, this model is a huge simplification; in general many other issues play a role as well, but it does show some of the optimization considerations that may play a role in a problem like this.

‖N.B. Don't forget to check (at least for yourself) that this leads to *maximal* capacity of the airport indeed.

**Actually, in reality the maximum weight of an airplane is different for take-off, during cruise flight and upon landing. This is particularly relevant as the fuel it carries adds significantly to the total weight of the airplane, so it's much heavier at the start of the flight than near the end. However, for the sake of simplicity, we do not consider such issues in this problem and will just consider one value for the maximum weight that the airplane can carry.

(f). Find out what the load of the aircraft would be to optimize the rate according to (19).

(g). What would be the relation between the empty weight w_0 and the maximum weight w_m for this optimum to be relevant? Consider the implications of these results on the use of designing ever-larger aircraft.

<u>Bonus</u>: You may want to investigate the problem for different waiting rules too:

(h). <u>Bonus</u>: Analyze the optimization problem if the waiting rule would be replaced by

$$\tau(w) = 1 + 6\,w^2, \tag{20}$$

instead of (17).

(i). <u>Bonus</u>: What can you say about the general optimization problem with any function $\tau(w)$ for the waiting time? Of course, it would be impossible to solve the problem explicitly in this case, but you can describe the equation you'd have to solve in general and try to draw some interesting *general* conclusions from that.

(j). <u>Bonus</u>: What could you say about the optimization problem if the waiting rule would be replaced by

$$\tau(w) = a\log(w) + b, \tag{21}$$

instead of (17), where a and b are some (positive) parameters. This would be a realistic formula for the waiting time if the induced turbulence is proportional to the wait of the aircraft and decays exponentially over time.
Warning: it would be impossible to solve the problem explicitly in this case, but it may still be possible to draw some interesting conclusions on how the results must depend on the parameters w_0, a and b. This would be rather advanced reasoning though!