

1. A structural engineer is studying the strength of aluminum alloy purchased from three vendors. Each vendor submits the alloy in standard-sized bars of 1.0, 1.5, or 2.0 inches. The processing of different sizes of bar stock from a common ingot involves different forging techniques, and so this factor may be important. Also, the bar stock is forged from ingots made in different heats. Each vendor submits two test specimens of each size bar stock from three randomly selected heat levels (these levels are different for each of the vendors).

|                     | <b>Vendor 1</b> |       |       | <b>Vendor 2</b> |       |       | <b>Vendor 3</b> |       |       |
|---------------------|-----------------|-------|-------|-----------------|-------|-------|-----------------|-------|-------|
| <b>Heat Level →</b> | 1               | 2     | 3     | 1               | 2     | 3     | 1               | 2     | 3     |
| <b>↓ Bar Size</b>   |                 |       |       |                 |       |       |                 |       |       |
| 1 inch              | 1.230           | 1.346 | 1.235 | 1.301           | 1.346 | 1.315 | 1.247           | 1.275 | 1.324 |
|                     | 1.259           | 1.400 | 1.206 | 1.263           | 1.392 | 1.320 | 1.296           | 1.268 | 1.315 |
| 1.5 inches          | 1.316           | 1.329 | 1.250 | 1.274           | 1.384 | 1.346 | 1.273           | 1.260 | 1.392 |
|                     | 1.300           | 1.362 | 1.239 | 1.268           | 1.375 | 1.357 | 1.264           | 1.265 | 1.364 |
| 2 inches            | 1.287           | 1.346 | 1.273 | 1.247           | 1.362 | 1.336 | 1.301           | 1.280 | 1.319 |
|                     | 1.292           | 1.382 | 1.215 | 1.215           | 1.328 | 1.342 | 1.262           | 1.271 | 1.323 |

- a) Write the linear statistical model for this experiment, and explain the model components. **{3 points}**
- b) Construct an ANOVA table including the expected mean squares in tabular form as shown below. State the null and alternative hypotheses for all effects, test each of the hypotheses, and interpret your results. Use  $\alpha = 0.05$ . **{4 points}**

| <b>Source</b> | <b>df</b> | <b>SS</b> | <b>MS</b> | <b>EMS</b> |
|---------------|-----------|-----------|-----------|------------|
|---------------|-----------|-----------|-----------|------------|

2. A researcher studied the effects of three experimental diets with varying fat contents on the total lipid (fat) level in plasma. Total lipid level is a widely used predictor of coronary heart disease. Fifteen male subjects who were within 20 % of their ideal body weight were grouped into five blocks according to age. Within each block, the three experimental diets were randomly assigned to the three subjects. Data on reduction in lipid level (in grams per liter) after the subjects were on the diet for a fixed period of time follow:

|   | Block        | Fat Content of Diet |       |       |
|---|--------------|---------------------|-------|-------|
|   | I            | J = 1               | J = 2 | J = 3 |
| 1 | Ages 15 – 24 | 0.73                | 0.67  | 0.15  |
| 2 | Ages 25 – 34 | 0.86                | 0.75  | 0.21  |
| 3 | Ages 35 – 44 | 0.94                | 0.81  | 0.26  |
| 4 | Ages 45 - 54 | 1.40                | 1.32  | 0.75  |
| 5 | Ages 55 - 64 | 1.62                | 1.41  | 0.78  |

- (a) Why do you think that age subject was used as a blocking variable?  
(3 points)
- (b) Obtain the analysis of variance table. Does it appear that the treatment means differ?  
(4 points)
- (c) Test whether or not the mean reductions in lipid level differ for the three diets; Use  $\alpha = 0.05$ . State the alternatives, decision rule, and conclusion. What is the p-value of the test?  
(4 points)
- (d) Estimate  $C_1 = \mu_{\bullet 1} - \mu_{\bullet 2}$  and  $C_2 = \mu_{\bullet 2} - \mu_{\bullet 3}$  using Bonferroni procedure with a 95 % family confidence coefficient. State your findings.  
(4 points)
- (e) A standard diet was not used in this experiment as a control. What justification do you think the experimenters might give for not having a control treatment here for comparative purposes?  
(2 points)
- (f) Based on the estimated efficiency measure, how effective was the use of the blocking variable as compared to a completely randomized design?  
(3 points)

3. A traffic engineer conducted a study to compare the total unused red light time for five different traffic light signal sequences. The experiment was conducted with a Latin square design in which the two blocking factors were (1) five randomly selected intersections and (2) five time periods. In the data table the five signal sequence treatments are shown in parentheses as A, B, C, D, E., and the numerical values are the unused red light times in minutes.

|                     | <b>Time Period</b> |          |          |          |          |
|---------------------|--------------------|----------|----------|----------|----------|
| <b>Intersection</b> | <b>1</b>           | <b>2</b> | <b>3</b> | <b>4</b> | <b>5</b> |
| <b>1</b>            | 15.2 (A)           | 33.8 (B) | 13.5 (C) | 27.4 (D) | 29.1 (E) |
| <b>2</b>            | 16.5 (B)           | 26.5 (C) | 19.2 (D) | 25.8 (E) | 22.7 (A) |
| <b>3</b>            | 12.1 (C)           | 31.4 (D) | 17.0 (E) | 31.5 (A) | 30.2 (B) |
| <b>4</b>            | 10.7 (D)           | 34.2 (E) | 19.5 (A) | 27.2 (B) | 21.6 (C) |
| <b>5</b>            | 14.6 (E)           | 31.7 (A) | 16.7 (B) | 26.3 (C) | 23.8 (D) |

- Write a linear model for this experiment, explain the terms, and compute the analysis of variance. (5 points)
- Compute the standard error for a signal sequence treatment mean and for the difference between two signal sequence treatment means. (3 points)
- Use the Multiple Comparisons with the Best procedure to select the set of signal sequences with the shortest unused red light time. (4 points)
- What is the relative efficiency of blocking by time periods? (4 points)

4. A horticulturalist conducted a nitrogen fertility experiment for lettuce in a randomized complete block design. Five rates of ammonium nitrate treatments (0, 50, 100, 150, and 250 lb/acre) were randomly assigned to each of two plots in each of two blocks for a total of four plots for each level of nitrogen. Each block consisted of ten plots, two plots for each treatment in each block. The data are the number of lettuce heads from each plot.

| Nitrogen | Block 1 |     | Block 2 |     |
|----------|---------|-----|---------|-----|
| 0        | 100     | 104 | 99      | 114 |
| 50       | 124     | 120 | 144     | 154 |
| 100      | 136     | 132 | 142     | 146 |
| 150      | 137     | 150 | 150     | 153 |
| 200      | 123     | 136 | 146     | 151 |

- (a) Write the linear model for the experiment, explain the terms, and compute the analysis of variance. Note that there are multiple plots for each treatment in each block. How does this affect your estimates of experiment error from the analysis of variance? (7 points)
- (b) Test the assumption of no block  $\times$  treatment interaction. (2 points)
- (c) Compute the linear and quadratic polynomial regression contrasts sum of squares partitions for nitrogen, and test the null hypotheses. Interpret the results. (4 points)
- (d) Are cubic deviations significant? (2 points)
5. A soil scientist conducted an experiment to evaluate the effects of soil compaction and soil moisture on the activity of soil microbes. Reduced levels of microbe activity will occur in poorly aerated soils. The aeration levels can be restricted in highly saturated or compacted soils. Treated soil samples were placed in airtight containers and incubated under conditions conducive to microbial activity. The microbe activity in each soil sample was measured as the percent increase in CO<sub>2</sub> produced above atmospheric levels.

The treatment design was a  $3 \times 3$  factorial with three levels of soil compaction (bulk density = mg soil/cubic meter) and three levels of soil moisture (kg water/kg soil). There were two replicate soil container units prepared for each treatment.

The CO<sub>2</sub> evolution/kg soil/day was recorded on three successive days. The data for each soil container unit are shown in the table.

| Density | Moisture | Unit | Day  |      |      |
|---------|----------|------|------|------|------|
|         |          |      | 1    | 2    | 3    |
| 1.1     | 0.10     | 1    | 2.70 | 0.34 | 0.11 |
|         |          | 2    | 2.90 | 1.57 | 1.25 |
|         | 0.20     | 3    | 5.20 | 5.04 | 3.70 |
|         |          | 4    | 3.60 | 3.92 | 2.69 |
|         | 0.24     | 5    | 4.00 | 3.47 | 3.47 |
|         |          | 6    | 4.10 | 3.47 | 2.46 |
| 1.4     | 0.10     | 1    | 2.60 | 1.12 | 0.90 |
|         |          | 2    | 2.20 | 0.78 | 0.34 |
|         | 0.20     | 3    | 4.30 | 3.36 | 3.02 |
|         |          | 4    | 3.90 | 2.91 | 2.35 |
|         | 0.24     | 5    | 1.90 | 3.02 | 2.58 |
|         |          | 6    | 3.00 | 3.81 | 2.69 |
| 1.6     | 0.10     | 1    | 2.00 | 0.67 | 0.22 |
|         |          | 2    | 3.00 | 0.78 | 0.22 |
|         | 0.20     | 3    | 3.80 | 2.80 | 2.02 |
|         |          | 4    | 2.60 | 3.14 | 2.46 |
|         | 0.24     | 5    | 1.30 | 2.69 | 2.46 |
|         |          | 6    | 0.50 | 0.34 | 0.00 |

- Describe the study in terms of the between-subjects and within-subjects designs. (5 points)
- Compute the mean of the observations for each soil bulk density and soil moisture level at each time of measurement, and make a profile plot of the results for each treatment. (5 points)
- Write a linear model for a split-plot analysis of variance, identify the terms, and indicate the assumptions necessary for the analysis with this model. (5 points)
- Conduct the split-plot analysis for the data, test the necessary hypotheses, and compute treatment means and their standard errors. What are your conclusions? (5 points)
- Obtain the residual plots from the split-plot analysis, and interpret them. (4 points)
- Compute the sum of squares partitions for the linear and quadratic, contrasts on time and their interactions with density and moisture treatments; test the null hypotheses, and interpret the results. (6 points)