Introduction to Statistics, Spring 2020 Final

Name/UNI

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Problem 1

Although there is some controversy regarding the appropriateness of IQ scores as a measure of intelligence, IQ scores are commonly used for a variety of purposes. One commonly used IQ scale has a mean of 100 and a standard deviation of 15, and scores are approximately normally distributed. If we define

x = IQ score of a randomly selected individual

a. What is the probability that a random person will have a score greater than 90? Between 90 and 110?
b. One way to become eligible for membership in Mensa, an organization purport- edly for those of high intelligence, is to have a Stanford-Binet IQ score above 130. What proportion of the population would qualify for Mensa membership?

Problem 2

The probability that a fluorescent bulb burns for at least 500 hours is 0.90. Of 8 such bulbs, find the probability that

a. All 8 burn for at least 500 hours.
b. At least one burns for at least 500 hours.
c. What is the expected value of the number of bulbs that burn for at least 500 hours? What is the variance of the number of bulbs that burn for at least 500 hours?
d. A store has 1000 fluorescent bulbs. Approximate the probability that at least 890 bulbs last 500 hours (use the continuity correction).
e. A store imported 60% of its fluorescent bulbs from China and 40% from Korea. It is known that 90% of the bulbs from China burns for at least 500 hours whereas it is 92% for the bulbs from Korea. A customer who bought a bulb from the store found that it last more than 500 hours. What is the probability that the bulb was imported from China?

Problem 3

An instructor knows from past experience that student examination scores have mean 77 and standard deviation 15. At present, the instructor is teaching two separate classes—one of size 81 and the other of size 121.

a. Approximate the probability that the average test score in the class of size 81 lies between 72 and 82.
b. Repeat (a) for the class of size 81.
c. Let $\bar{X}_{81}$ and $\bar{Y}_{121}$ be the average test score in the class of 81 and 121, respectively. What is the expected value and variance of $\bar{X}_{81} - \bar{Y}_{121}$? What is the distribution of $\bar{X}_{81} - \bar{Y}_{121}$?
d. What is the approximate probability that the average test score in the class of size 81is higher than that in the class of size 121?

Problem 4

In the “3 Spot” version of the former California Keno lottery game, the player picked three numbers from 1 to 40. Ten possible winning numbers were then randomly selected. It cost $1 to play. The table shows the possible outcomes.

Number.of.Matches	**Amount.Won**	**Probability**
3	$20	0.012
2	$2	1.370
0 or 1	$0	0.851

a. Compute the expected value of the amount won for this game. Interpret what it means.
b. Calculate the variance of this game.
c. A customer buys 5 tickets. Calculate the expected value and variance of the **net** gain

Problem 5

A television manufacturer claims that 90% of its TV sets will need no service during the first 3 years of operation. A consumer agency wishes to check this claim, so it obtains a random sample of n= 100 purchasers and found that 86 did not have the TV set repaired during the first 3 years after purchase.

a. Test the manufacturer claim at the 0.05 level.
b. What is the p-value for the test?

Problem 6

All the students at a certain country are to be given a psychological task. To determine the average time it will take a student to perform this task, a random sample of 36 students was chosen and each was given the task. If it took these students an average of 12.4 minutes to complete the task with a sample standard deviation of 3.0 minutes.

a. Find a 95% confidence interval estimate for the average time it will take all students in the school to perform this task.
b. **Explain** what happens to the width of the confidence interval when
   - the sample size increases
   - the confidence level is reduced to 90%

Problem 7

The paper *"If It's Hard to Read, It's Hard to Do" (Psychological Science [2008]: 986–988)* described an interesting study of how people perceive the effort required to do certain tasks. Each of 20 students was randomly assigned to one of two groups. One group was given instructions for an exercise routine that were printed in an easy-to-read font (Arial). The other group received the same set of instructions, but printed in a font that is considered difficult to read (Brush). After reading the instructions, subjects estimated the time (in minutes) they thought it would take to complete the exercise routine. Summary statistics are given below.

	Easy font	*Difficult font*
n	10	10
$\bar{x}$	8.23	14.10
s	5.61	9.28

Construct a 90% confidence interval for the difference, assuming that the degree of freedom for the sample mean difference is ~14.

Problem 8

Suppose a shipment of oranges is advertised to weigh 5 pounds per bag. We know that not every bag can contain exactly 5 pounds of oranges. We decide to take a random sample of 100 bags of oranges and find out what they tell us about the population of all bags in this shipment. We are only interested in whether or not the bags are underweight, so each bag is weighed and counted as underweight if it weighs less than 5 pounds. Five bags in our sample of 100 were found to be underweight.

a. Construct a 90% confidence interval for the true proportion of bags that are underweight.
b. What is the 95% margin error?
c. What is the interpretation of the confidence interval?

Problem 9

In the population, it is hypothesized that flags have a mean usable life of 100 days. Twenty-five flags are flown in the city of Tuscaloosa and are found to have a sample mean usable life of 200 days with a standard deviation of 216 days. Does the sample mean in Tuscaloosa differ from that of the population mean?

a. Conduct a two-tailed test at the .01 level of significance.
b. Construct a 99% confidence interval.

Problem 10

Some people seem to believe that you can fix anything with duct tape. Even so, many were skeptical when researchers announced that duct tape may be a more effective and less painful alternative to liquid nitrogen, which doctors routinely use to freeze warts. The article *"What a Fix-It: Duct Tape Can Remove Warts" (San Luis Obispo Tribune, October 15, 2002)* described a study conducted at Madigan Army Medical Center. Patients with warts were randomly assigned to either the duct tape treatment or the more traditional freezing treatment. Those in the duct tape group wore duct tape over the wart for 6 days, then removed the tape, soaked the area in water, and used an emery board to scrape the area. This process was repeated for a maximum of 2 months or until the wart was gone. Data consistent with values in the article are summarized in the following table:

Treatment	*n*	*Number of Wart Successfully removed*
Liquid nitrogen freezing	100	60
Duct tape	104	88

Do these data suggest, at the 0.05 level, that freezing is less successful than duct tape in removing warts?

Problem 11

Do children diagnosed with attention deficit/hyperactivity disorder (ADHD) have smaller brains than children without this condition? This question was the topic of a research study described in the paper *"Developmental Trajectories of Brain Volume Abnormalities in Children and Adolescents with Attention Deficit/Hyperactivity Disorder" (Journal of the American Medical Association [2002]: 1740–1747)*. Brain scans were completed for 152 children with ADHD and 139 children of similar age without ADHD. Summary values for total cerebral volume (in milliliters) are given in the following table:

	n	$\bar{X}$	s
Children with ADHD	152	1059.4	117.6
Children without ADHD	139	1104.5	111.3

Assume that the 2 population sample standard deviations are equal. Test the above hypothesis at the 0.05 level.

Problem 12

To determine if chocolate milk was as effective as other carbohydrate replacement drinks, nine male cyclists performed an intense workout followed by a drink and a rest period. At the end of the rest period, each cyclist performed an endurance trial in which he exercised until exhausted and time to exhaustion was measured. Each cyclist completed the entire regimen on two different days. On one day the drink provided was chocolate milk and on the other day the drink provided was a carbohydrate replacement drink. Data consistent with summary quantities appearing in the paper *"The Efficacy of Chocolate Milk as a Recovery Aid" (Medicine and Sci- ence in Sports and Exercise [2004]: S126)* appear in the table below.

a. Is there evidence that the mean time to exhaustion is greater after chocolate milk than after carbohydrate replacement drink? Use a significance level of .05.
b. Construct a 90% confidence interval for the difference in exhaustion times.

Exhaustion time (min)

Cyclist	1	2	3	4	5	6	7	8	9	$\bar{X}$	*s*
Chocolate Milk	24.85	50.09	38.30	26.11	36.54	26.14	36.13	47.35	25.08	34.51	9.69
Carbohydrate Replacement	10.02	29.96	37.40	15.52	9.11	21.58	31.23	22.04	17.02	21.54	9.75
Difference	14.83	20.13	0.90	10.59	27.43	4.56	4.90	25.31	8.06	12.97	9.53