

The coordinates of  $P_0 = (0, 0)$  and of  $P_4 = (1020, 0)$  are known but the coordinates of the points  $P_1$ ,  $P_2$  and  $P_3$  must be determined.

To determine the coordinates of a point  $P$ , measure the distances from  $P = (x_P, y_P)$  to two points with known coordinates,  $A = (x_A, y_A)$  and  $B = (x_B, y_B)$ .

If we call the distances between  $P$  and  $A$  and  $P$  and  $B$   $L_A$  and  $L_B$  respectively, we get the following equation system where the equations describe two circles with centers in  $A$  and  $B$  and with radii given by  $L_A$  and  $L_B$ .

$$\sqrt{(x_P - x_A)^2 + (y_P - y_A)^2} = L_A \quad (1)$$

$$\sqrt{(x_P - x_B)^2 + (y_P - y_B)^2} = L_B. \quad (2)$$

This system of equations has two solutions (intersection points) where there are two points,  $P$  and  $Q$ , which are both at the same distance from points  $A$  and  $B$ . In this case it is point  $P$ , the right intersection we are interested in. See Figure 2.

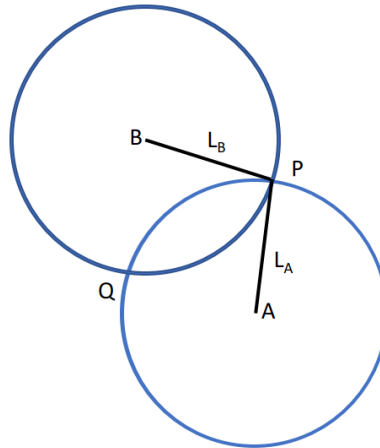


Figure 2: Determination of coordinates for the point  $P$  with binding. The coordinates of the points  $A$  and  $B$  and the lengths  $L_A$  and  $L_B$  are known.

**1a)** Start by determining the coordinates (with the appropriate tolerance) for points P1, P2 and P3 using binding. For each point, a non-linear system of equations must correspond (1) - (2) is solved by Newton's method. Initial values are determined by drawing the circles.

The table below shows points A and B with known coordinates and measured distances between A and P and B and P for points P1, P2 and P3.

The coordinates are given in meters from the point (0, 0).

The program should not use code repetition. Use a **for-loop** for the three points. **Report the coordinates and that Newton's method converges squarely.**

$P$	$A = (x_A, y_A)$	$B = (x_B, y_B)$	$L_A$ [m]	$L_B$ [m]
$P_1$	(175, 950)	(160, 1008)	60	45
$P_2$	(410, 2400)	(381, 2500)	75	88
$P_3$	(675, 1730)	(656, 1760)	42	57

**1b)** Determine the fourth degree polynomial,  $p(x)$ , which passes through the five points P0, P1, P2, P3 and P4. **Draw the path (graph of the polynomial) for  $x \in (0, 1020)$ . Also draw the five the interpolation points and mark them with 'o'. Report the coefficients in the polynomial and the figure with the plotted path and the interpolation points.**

**1c)** Calculate the length of the new road from the point (0, 0) to the point (1020, 0). Use the trapezoidal rule with the appropriate step length. **Report the length of the road and that your implementation of the trapezoidal rule has an accuracy order 2. Tip: how do you calculate the arc length of a curve?**

**1d)** Assume the lengths  $L_A$  and  $L_B$  have an uncertainty of  $\pm 5$  m. **Calculate the uncertainty length on the road with experimental disturbance calculation. Report the uncertainty in the length calculation.**

**Tip:** For the experimental disturbance count, the code that solves a) -c) is packaged in a **"black box "** (a Matlab function) that takes  $L_A$  and  $L_B$  for the three points that input and gives the length of the path as output.