Exercise 1

Consider the following linear model:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$$

[a] Specify under what conditions we can identify the parameters of the model. Be very specific and write only the conditions needed.

[b] Assume that x_3 is an endogenous variable, that is $\mathbb{E}[x_3 u] \neq 0$. What are the implications of x_3 being an endogenous regressor?

[c] One of your classmates, Ana, tells you that you can use an instrument, z, for the endogenous variable. What are the assumptions required for z to be an instrument for x_3. Show mathematically, how you will conduct the IV and 2SLS procedure? Be very specific and layout the steps in a clear way.

[d] In the 2SLS approach explain how will you test the relevance assumption of the instrument? State the null and alternative hypothesis. Also, explain how can we test endogeneity. State the null and alternative hypothesis.

Now consider the following model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + u$$

[e] Set up the problem of minimizing the sum of squared residuals, derive the first order conditions and obtain the least square estimates $\hat{\beta}_0, \hat{\beta}_1$, and $\hat{\beta}_2$. Make sure to show all the steps very clearly.

Now consider that one of your classmates tells you that x_2 is an endogenous variable and proposes an instrument z, and argues that $Cov(z, u) = 0$ and $Cov(z, x_2) \neq 0$. Your classmate proposes a two-stage least square (2SLS) procedure as follows:

$$x_2 = \gamma_0 + \gamma_2 z + u$$
$$y = \beta_0 + \beta_1 x_1 + \beta_2 \hat{x}_2 + u$$

[f] Is the procedure proposed above correct? Explain why or why not.

Exercise 2

Suppose that the demand function of a good is given by $q = \gamma_0 + \gamma_1 p + u$, where p represents the price of the good, q represents the quantity of the good, and u represents the unobserved factors that determine the demand function. Also, let the supply of the good be, $q = \delta_0 + \delta_1 p + \eta$, where η captures the unobserved factors that determine the supply function. Let $\mathbb{E}[u] = \mathbb{E}[\eta] = 0$, $Var(u) = \sigma_u^2$, and $Var(\eta) = \sigma_\eta^2$. In addition assume that the unobserved components, u and η, are not correlated, that is $Cov(u, \eta) = 0$.

[a] Solve the system of equations (demand and supply) and show that p and q depend on the unobserved components u and η.

[b] Obtain the means of the price and quantity derived in part [a].

[c] Obtain the variance of the price and quantity derived in part [a].

Now let $\{(q_i, p_i) : i = 1, 2, \ldots, N\}$ be a random sample and we regress q_i on p_i.

(i) Use your results in part [b] and [c] to derive the estimates of the regression.

(ii) An economist uses the estimate of γ_1 as the slope of the demand function. Is the estimated value of γ_1 too large or too small? Show and explain in detail clearly. [Remember that the demand is downward and the supply is upward.]

Now suppose that you have the following Cobb-Douglas production function for a firm:

$$y_i = A_i L_i^\alpha K_i^\beta$$

where L_i denotes labor, K_i denotes capital, and A_i is technology. Assume that $A_i = e^{\beta_0 + \beta_1 x_i + u_i}$, where x_i denotes an observed firm characteristic and u_i denote the firm's unobserved characteristics. Our parameters of interest are β_1, α and β.

[d] Rewrite the Cobb-Douglas production function in a linear form, that is the parameters must enter the model linearly. [Hint: remember the log transformation from intermediate micro class.]

[e] Write down in a clear way the identification conditions required for the model parameters to be identified.

[f] How would you test the joint significance of α and β? Show the steps in a clear way.