New Keynesian Model

This section, we develop a dynamic stochastic general equilibrium model with monopolistic competition and nominal price rigidities, which can form the basis for a simple linear macroeconomic model that is useful for policy analysis. We combine a stochastic MIU model with the assumption of monopolistically competitive goods markets and Calvo-style price stickiness. In this model, nominal wages will be allowed to fluctuate freely. The model is a consistent general equilibrium model in which all agents face well-defined decision problems and behave optimally. This modification yields a framework, often referred to as New Keynesian, that is directly linked to the more traditional aggregate supply-demand (AS-IS-LM) model that long served as one of the workhorses for monetary policy analysis.

1 The New Keynesian Model: Main Elements and Features

In the 1970s, 1980s, and early 1990s, the standard models used for most monetary policy analysis combined the assumption of nominal rigidity with a simple structure linking the quantity of money to aggregate spending. This linkage was usually directly through a quantity theory equation in which nominal demand was equal to the nominal money supply, often with a random disturbance included, or through a traditional textbook IS-LM model. While the theoretical foundations of these models were weak, the approach proved remarkably useful in addressing a wide range of monetary policy topics. More recently, attention has been placed on ensuring that the model structure is consistent with the underlying behavior of optimizing economic agents. The standard approach today builds on a dynamic, stochastic, general equilibrium framework based on optimizing behavior, combined with some form of nominal wage and/or price rigidity. This modification yields a framework, often referred to as New Keynesian. Early examples of models with these properties include those of Yun (1996), Goodfriend and King (1997), Rotemberg and Woodford (1995, 1997), and McCallum and Nelson (1999).

The new Keynesian monetary model inherited its core from the RBC model and its dynamic stochastic general equilibrium (DSGE) structure. These are the assumption of (i) an infinitely-lived representative household, who seeks to maximize the utility from consumption and leisure, subject to an intertemporal budget constraint, and (ii) a large number of firms with access to an identical technology, subject to exogenous random shocks. Also, as in RBC theory, an equilibrium takes the form of a stochastic process for all the economy's endogenous variables, that come from optimal intertemporal decisions by households and firms, given their objectives and constraints, and with the clearing of all markets.

The new Keynesian modelling approach, however, depart from those found in classical monetary models. Here is a list of some of the key elements and properties of the resulting models:

- Monopolistic competition: The prices of goods and inputs are set by firms in order to maximize their objectives.
- Nominal rigidities: Firms are subject to some constraints on the frequency with which they can adjust the prices of the goods and services they sell. The same kind of friction applies to workers in the presence of sticky wages.
- Endogenous variations in the capital stock are ignored. This follows McCallum and Nelson (1999), who show that, at least for the United States, there is little relationship between the capital stock and output at business-cycle frequencies.
- Short run non-neutrality of monetary policy: As a consequence of the presence of nominal rigidities, changes in short term nominal interest rates are not matched by one-for-one changes in expected inflation, thus leading to variations in real interest rates. The latter bring about changes in consumption and investment and, as a result, on output and employment. In the long run, however, all prices and wages adjust, and the economy reverts back to its natural equilibrium.

Therefore, the non-neutrality of monetary policy resulting from the presence of nominal rigidities makes room for potentially welfare-enhancing interventions by the monetary authority, in order to minimize the existing distortions (Imperfect competition and nominal rigidities).

2 The specification of the model

The model includes households, firms and a government sector: Households consume, supply labor, hold real money balances, and bonds (save). Producers are monopolistic competitors that each produce a differentiated product using labor. These firms set nominal prices on a staggered basis.

Monopolistically competitive firms The economy is composed of a continuum of a monopolistically competitive firm indexed by $j \in [0, 1]$, whose total is normalized to unity. Each firm produces a differentiated good, $Y_t(j)$, which enters the aggregate consumption basket, and demands labor.

Although each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{1}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time. Furthermore, it is assumed that $A_t = A_{t-1}^{\rho_a} e^{\varepsilon_t^a}$, and $0 < \rho_a < 1$, $\varepsilon_t^a \sim i.i.dN(0, \sigma_a^2)$. Monopolistic firm also sets price of its own good, $P_t(j)$.

Households There are also continuum of infinitely-lived households indexed by $h \in [0,1]$. Each household, however, is identical in a sense that they have same preference and is subject to the same constraint. Since households are identical we drop the h subscript and impose symmetric condition on the households's behavior. Then we can only consider the representative household. Each of them consumes a basket of differentiated goods and supplies labor. The life-time utility of the household h is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{1-\sigma} C_t^{1-\sigma} + \frac{1}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{1}{1+\varphi} N_t^{1+\varphi} \right\},$$

where $0 < \beta < 1$ is the discount factor, $\sigma > 0, \nu > 0, \varphi > 0$, (the inverse of Frisch elasticity of labor supply). C_t is a basket of differentiated goods defined as

$$C_t = \int_0^1 C_t(j) dj, \tag{2}$$

where $C_t(j)$ is the household's demand for good j.

Households can hold savings in the form of money balances and bonds. Households also own the firms. So each household has claims to any profit of firm z. Thus, the budget constrain for the household h is

$$\int_0^1 P_t(j) C_t(j) di + M_t + Q_t D_t = D_{t-1} + M_{t-1} + W_t N_t + TR_t + \int_0^1 \Pi_t(j) dj,$$

for $t = 0, 1, 2, ...,$where W_t denotes nominal wage, D_t represents the quantity of one-period, nominally riskless discount bonds purchased in period $t-1$ and maturing in period t. Each bond pays one unit of money at maturity and its price is Q, then nominal interest rate, i_t is $\frac{1-Q_t}{Q_t}$. Thus purchasing nominal bonds is a way of saving. M_t is the quantity of nominal balances accumulated during time $t-1$ and carried over into time t. Notice that $TR_t(h)$ and $\Pi_t(j)$ are government transfer and a profit of firm j received by household, respectively.

The riskless short-term nominal interest rate i_t corresponds to the solution the equation

$$\frac{1}{1+i_t} = Q_t$$

The Fisher Parity

$$1 + i_t = E_t \left\{ R_t \frac{P_{t+1}}{P_t} \right\},$$

where $\frac{P_{t+1}}{P_t} = 1 + \pi_{t+1}$ is the gross inflation rate. R_t is the gross real return from bonds holding. The nominal interest rate is discounted for the gross inflation rate.

The Consumption-Based Price Index The consumption-based price index P_t tells how much real consumption C_t the consumer derives from a given nominal expenditure EX_t. Then:

$$P_t = \left[\int_0^1 P_t(j) dj \right] \tag{3}$$

Then, we can write $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$

3 Household

The representative household chooses $\{C_t, N_t, M_t, D_t\}_{t=0}^{\infty}$ to maximize

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} C_t^{1-\sigma} + \frac{1}{1-\nu} \left(\frac{M_t}{P_t} \right)^{1-\nu} - \frac{1}{1+\varphi} N_t^{1+\varphi} \right] \right\},$$

subject to

$$P_t C_t + M_t + Q_t D_t = D_{t-1} + M_{t-1} + W_t N_t + TR_t + \int_0^1 \Pi_t(j) dj,$$

The first-order conditions are

$$C_t^{-\sigma} = \lambda_t P_t, \tag{4}$$

$$\left(\frac{M_t}{P_t} \right)^{-\nu} \frac{1}{P_t} + \beta E_t \{\lambda_{t+1}\} = \lambda_t, \tag{5}$$

$$N_t^{\varphi} = \lambda_t W_t, \tag{6}$$

$$Q_t \lambda_t = E_t \{\lambda_{t+1}\}, \tag{7}$$

$$TVC : \lim_{t \to \infty} E_0 \left\{ \beta^t \lambda_t M_t \right\} = \lim_{t \to \infty} E_t \left\{ \beta^t \lambda_t D_t \right\} = 0.$$

Updating the envelope conditions (4) one period forward and using it into the FOCs (5), (6) and (7) we get

$$C_t^{-\sigma} = \beta E_t \{ R_t C_{t+1}^{-\sigma} \}, \tag{8}$$

$$C_t^{-\sigma} = (\frac{M_t}{P_t})^{-\nu} + \beta E_t \{ \frac{P_t}{P_{t+1}} C_{t+1}^{-\sigma} \}, \tag{9}$$

and

$$N_t^{\varphi} = C_t^{-\sigma} \frac{W_t}{P_t}, \tag{10}$$

where

$$R_t = (1 + i_t) E_t \left\{ \frac{P_t}{P_{t+1}} \right\},$$

Equation (8) is a standard Euler equation. Equation (9) is the dynamic condition for the choice of money holdings. The marginal cost of foregoing one unit of consumption today must be equal to the pecuniary benefit of being able to buy an extra unit of consumption tomorrow, plus the nonpecuniary benefit measured by the current utility flow of an extra unit of money. Using (8), we can write (9) as follows:

$$\frac{\left(\frac{M_t}{P_t} \right)^{-\nu}}{C_t^{-\sigma}} = \frac{i_t}{1 + i_t} \tag{11}$$

which defines the money demand function. Equation (10) is an intratemporal condition capturing the consumption/leisure trade-off. It has the interpretation that the marginal rate of substitution between consumption and leisure be equal to the real wage.

4 Government

There is no government spending. However, government gives a lump-sum transfer to household and finance this through seignorage revenue. Thus government budget constraint is

$$TR_t = M_t - M_{t-1}.$$

It is assumed that government (or central bank) creates money, and creates money through the following process

$$M_t = M_{t-1}^{\rho^m} e^{\varepsilon_t^m}, \tag{12}$$

where $0 < \rho^m < 1$ and ϵ_t^m is a stochastic component of money supply, interpreted as growth rate of money supply, which cannot be observable by the public (household and firms). It is assumed that$\varepsilon_t^m \sim_{i,i,d} N(0, \sigma_m^2)$. Here we do not introduce any specific money creation process.

4.1 Taylor's Formula

Most central banks use a (short-term) interest rate as their policy instrument, and the goal of monetary policy is to stabilize output and in inflation. When shocks push output and inflation away from their long-run levels, the central bank adjusts the interest rate to push them back. Therefore, we can assume that the central bank run monetary policy in interest-rate targeting instead. In 1993, in a paper by John Taylor, central bank's policy based on the Taylor rule is proposed. Taylor express the central bank's interest rate rule in a simple equation.

$$1 + i_t = \left(\frac{Y_t}{Y_t^n}\right)^{\phi_y} \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} e^{v_t}$$

where Y_t^n is output at the natural level, $\Pi_t = P_t/P_{t-1}$ is an inflation rate, ϕ_y, ϕ_π are positive constants, and v_t is an interest rate shock with $\sim_{i,i,d} N(0, \sigma_v^2)$. Under the interest rate targeting policy, interest rate is the policy instrument, and IS curve can be solved directly for output. That is, the money market condition is no longer needed, although it will determine the level of nominal money balance necessary to ensure money market equilibrium. By fixing the rate of interest, the central bank lets the money stock adjust endogenously to equal the level of money demand given by the interest rate and the level of income.

5 Firm

5.1 Optimal price decision when price is flexible

Now let us consider the monopolistic firm's problem when it does not face any restriction to change its price. Each firm produces a differentiated good, but they all use an identical technology, represented by the production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{13}$$

where A_t represents the level of technology, assumed to be common to all firms and to evolve exogenously over time. All firms face an identical demand schedule, and take the aggregate price level P_t and aggregate consumption index C_t as given.

The firm maximizes the discounted stream of profits returned to the household. The monopolistic firm chooses $P_t(j)$ to solve the firm's profit at time t

$$\Pi_t(j) = P_t(j)Y_t(j) - TC_t(j)$$

where TC_t is a firm's total cost function, that is $W_tN_t(j)$. The resulting FOC is

$$P_t(j) = \mu MC_t^N(j). \tag{14}$$

where $\mu > 1$ is the mark-up. Since $MC_t^N(i) = \frac{1}{1-\alpha}W_tA_t^{1/1-\alpha}Y_t(j)^{\alpha/1-\alpha}$,

$$P_t(j) = \mu\frac{1}{1-\alpha}W_tA_t^{1/1-\alpha}Y_t(j)^{\alpha/1-\alpha}. \tag{15}$$

is labor demand curve for firm j.

Notice that

$$\begin{aligned} MC_t^N &= \frac{\partial TC_t}{\partial Y_t} = \frac{\partial TC_t/\partial N_t}{\partial Y_t/\partial N_t} \\ &= \frac{1}{MPN_t}\frac{\partial TC_t}{\partial N_t} \end{aligned}$$

5.2 Staggered long-term pricing

Now let us consider the monopolistic firm's problem when they cannot adjust their price freely. The specific model of price stickiness is due to Rotemberg (1982) (Quadratic Costs of Price Changes). Rotemberg modeled the sluggish adjustment of prices by assuming that firms faced quadratic costs of making price changes. Rotemberg model assumed all firms could adjust their price each period, but because of adjustment costs, they would only close partially any gap between their current price and the optimal price. Firm i is a monopolist on its market and sets the log price, $p_t(j)$, to maximize the value of the firm: the expected discounted sum of profits. If there were no costs of adjusting this price, then the price would be equal to some value, $P_t^*(j)$, which we call the flexible price optimum. With costs of adjusting the price model formulate the maximization problem in two steps. First, find the flexible price optimum, $P_t^*(j)$. Second, minimize the loss from not being at $P_t^*(j)$, and from incurring adjustment costs.

Suppose, for example, that the desired price of firm j depends on the aggregate average price level and a measure of real economic activity. We assume that firm's desired price is given by (5.2) as

$$P_t^*(j) = \mu MC_t^N(j) = \mu MC_t(j)P_t.$$

where $MC = MC^N/P$ is the real marginal cost. Then in log-linear term

$$p_t^*(j) = mc_t(j) + p_t \tag{16}$$

where p_t is the log aggregate price level and mc_t is the real marginal cost. Notice that if all firms are identical, as is assumed here, $p_t^* = p_t$ for all j, and (16) can be written as $p_t^* = p_t + mc_t$.

Furthermore, assume profits loss are a decreasing quadratic function of the deviation of the firm's actual log price from $p_t^*(j)$:

$$L_t(j) = -\delta \left[p_t(j) - p_t^*(j)\right]^2 = -\delta \left[p_t(j) - p_t - mc_t\right]$$

The costs of adjusting price are also quadratic and equal to

$$\mathcal{C}_t(j) = \phi \left[p_t(j) - p_{t-1}(j)\right]^2$$

If $\mathcal{C} = 0$ (no adjustment cost), then the firm will always set its actual price equal to the unrestricted optimal price.

Each period,firm i chooses $p_t(j)$ to maximize

$$\sum_{t=0}^{\infty} \beta^t E_0\{\Pi_t(j) - C_t(j)\}$$

where $0 < \beta < 1$ is a subjective discount rate. The first-order condition for the firm's problem is

$$-\delta \left[p_t(i) - p_t^*(j)\right] - \phi \left[p_t(j) - p_{t-1}(j)\right] + \beta\phi \left[E_t p_{t+1}(j) - p_t(j)\right] = 0$$

Since all firms are identical, $p_t(i) = p_j, \forall i$, and one can rewrite this first-order condition in terms of inflation as

$$\pi_t = \beta E_t \pi_{t+1} + \frac{\delta}{\phi} mc_t$$

Inflation depends on the real activity variable mc_t and expected future inflation. Because firms are concerned with their price relative to other firms' prices, and they recognize that future price changes are costly, the price they set at time t is higher if they anticipate higher inflation in the future. Solving (5.2) forward, inflation is expressed as a function of the present discounted value of current and future deviations of real marginal costs from its steady state

$$\pi_t = \frac{\delta}{\phi} \sum_{k=0}^{\infty} \beta^k E_t \left\{mc_{t+k}\right\}$$

6 Flexible price Equilibrium and the steady state

6.1 Symmetric monopolistic competitive equilibrium

Equilibrium in goods market requires that the quantity produced of each good matches the quantity demanded. Thus, in equilibrium,

$$C_t(j) = Y_t(j)$$

for all $j \in [0,1]$ and all t. Letting aggregate output be defined as

$$Y_t \equiv \int_0^1 Y_t(j)dj$$

it follows that

$$Y_t = C_t$$

must hold for all t.

Every good is equally important in household's utility. Every producers has the identical technology in production. In equilibrium under flexible prices, all monopolistic producers behave identically to each other in the sense that they produce same level of output and charge the same price. Thus a symmetric equilibrium is characterized by the following conditions

$$P_t(j) = P_t, \forall j,$$
$$N_t^d(j) = N_j^d, \forall j,$$

Then, aggregate price level is P_t.

Aggregate employment is given by the sum of employment across firms:

$$N_t = \int_0^1 N_t(j)dj$$

Using (1)

$$N_t = \int_0^1 \left(\frac{Y_t(j)}{A_t}\right)^{\frac{1}{1-\alpha}} dj$$

Then,

$$Y_t = A_t N_t^{1-\alpha} \tag{17}$$

Imposing the symmetry condition into the pricing (or labor demand) equation for the differentiated good z, noticing $MC_t^N = \frac{W_t}{(1-\alpha)A_t N_k^{-\alpha}}$ yields the following labor demand as

$$P_t = \mu \frac{W_t}{(1-\alpha)A_t N_t^{-\alpha}} \tag{18}$$

Since everybody is indifferent between borrowing and lending, in equilibrium the supply of bonds is zero, so

$$D_t = 0. \tag{19}$$

Equation (19) is a bonds market clearing condition. Since every individual is same in this economy, aggregate saving is equal to zero in equilibrium.

6.2 Equilibrium conditions

The equations characterizing equilibrium in the resulting model consists of two blacks: Aggregate demand and supply blocks. Replacing $C_t = Y_t$, aggregate demand block is given as

$$Y_t^{-\sigma} = \beta E_t\{R_t Y_{t+1}^{-\sigma}\}, \text{ (IS)} \tag{20}$$

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\nu}}{Y_t^{-\sigma}} = \frac{i_t}{1+i_t}, \text{ (Money demand)} \tag{21}$$

$$M_t = M_{t-1}^{\rho^m} e^{\varepsilon_t^m}, \text{ (Money supply process)} \tag{22}$$

Combining (21) and (36) yields the money market equilibrium condition or LM curve. However, LM curve can be replaced with the following Taylor rule

$$1 + i_t = \left(\frac{Y_t}{Y_n}\right)^{\phi_y} (\Pi_t/\Pi)^{\phi_\pi} e^{v_t}, \text{ (Taylor Rule)} \tag{23}$$

where $1 + i_t = E_t\left\{R_t \frac{P_{t+1}}{P_t}\right\}$ by the Fisher Parity.

The aggregate supply block is

$$Y_t = A_t N_t^{1-\alpha}, \text{ (Aggregate Production)} \tag{24}$$

$$N_t^{\varphi} = Y_t^{-\sigma}\frac{W_t}{P_t}, \text{ (Labor supply)} \tag{25}$$

$$P_t = \mu MC_t^N \text{ Labor demand)} \tag{26}$$

$$MC_t^N = \frac{W_t}{(1-\alpha)A_t N_t^{-\alpha}} \tag{27}$$

$$A_t = A_{t-1}^{\rho_a} e^{\varepsilon_t^a}, \text{ (Technology)} \tag{28}$$

6.3 the equilibrium under the flexible prices

Now let us discuss the equilibrium under the flexible prices. The aggregate supply side of the economy is characterized by the following equations representing aggregate labor market. Combining the labor supply, (25),with the aggregate production (**??**), we have

$$A_t^{\sigma} N_t^{\varphi+\sigma(1-\alpha)} = \frac{W_t}{P_t} \tag{29}$$

Combines the labor demand curve (18) and the labor supply curve (29), giving the equilibrium labor as

$$N_t = (1-\alpha)^{\frac{1}{\varphi+\sigma(1-\alpha)+\alpha}} A_t^{\frac{1-\sigma}{\varphi+\sigma(1-\alpha)+\alpha}} = N_t^n. \tag{30}$$

Equation (30) represents a natural level of employment since it is an equilibrium level of employment under the flexible price. Then the natural rate of output is given by

$$Y_t^n = (1-\alpha)^{\frac{1-\alpha}{\varphi+\sigma(1-\alpha)+\alpha}} A_t^{\frac{1+\varphi}{\varphi+\sigma(1-\alpha)+\alpha}} \tag{31}$$

A higher markup means a lower output in equilibrium, implying a lower real wage and a lower labor supply. Equation (31) shows that the technological change will affect the natural rate of output.

From the Euler equation (also known as IS curve), we can have an expression for real interest at the natural level as

$$R_t^n = \beta^{-1} E_t \left\{ \left(\frac{A_{t+1}}{A_t} \right)^{\frac{\sigma(1+\varphi)}{\varphi+\sigma(1-\alpha)+\alpha}} \right\} \tag{32}$$

In the money market equilibrium, price level is determined as

$$P_t^n = M_{t-1}^{\rho_m} e^{\varepsilon_t} (Y_t^n)^{-\frac{\sigma}{\nu}} \left(\frac{i_t^n}{1+i_t^n} \right)^{\frac{1}{\nu}}$$

where $1 + i_t^n = E_t \left\{ R_t^n \frac{P_{t+1}^n}{P_t^n} \right\}$ is the nominal interest rate at the natural level.

6.4 The zero inflation steady state

At the steady state, $A_{t+1} = A_t = A_{ss} = 1$, and $\varepsilon_t^a = 0$. Then from the Euler equation (also known as IS curve):

$$R_{ss} = \frac{1}{\beta}. \tag{33}$$

The steady-state level of employment is

$$N_{ss} = (1-\alpha)^{\frac{1}{\varphi+\sigma(1-\alpha)+\alpha}}.$$

and

$$Y_{ss} = (1-\alpha)^{\frac{1-\alpha}{\varphi+\sigma(1-\alpha)+\alpha}}$$

In the money market equilibrium, price level is determined as

$$P_{ss} = M_{ss}^{\rho_m} e^{\varepsilon_t} (Y_t^n)^{-\frac{\sigma}{\nu}} \left(\frac{i_t^n}{1+i_t^n} \right)^{\frac{1}{\nu}}$$

At the zero inflation steady state $\varepsilon_t^m = 0$, and the real money balances are constant over time, which implies

$$\frac{M_t}{P_t} = \frac{M_{t-1}}{P_{t-1}}$$
$$\frac{M_t}{M_{t-1}} = \frac{P_t}{P_{t-1}}$$

Thus, at the steady state

$$\pi_{ss} = 0$$

From the Fisher Parity and (33), in the flexible price steady state equilibrium, the gross nominal interest rate is equal to:

$$1 + i_{ss} = \frac{1}{\beta}$$

7 The Short-Run Dynamics under Sticky Price

Now we study the equilibrium dynamics in the short-run, where prices are sticky, but wage is flexible. It is assumed that firms adjust their prices infrequently. First, we look the short-run dynamics of aggregate demand. Then we derive the short-run dynamics for aggregate supply side.

7.1 Aggregate demand block

The aggregate demand side of the economy consists of three equations: IS curve and money demand and supply curves which consists of LM curve. Log-linearization of (20), (36), and (21) yield the following linear equations:

$$E_t\{y_{t+1}\} - y_t = \sigma^{-1} r_t \tag{34}$$

Rearranging this equation using log-liner version of Fisher parity ($i_t = r_t + E_t\{\pi_{t+1}\}$), we have

$$y_t = E_t\{y_{t+1}\} - \sigma^{-1}\left[i_t - E_t\{\pi_{t+1}\}\right], \text{ (Intertemporal IS curve)}$$

and

$$m_t - p_t = a_c y_t - a_i i_t \tag{35}$$

with $a_c = \frac{\sigma}{\nu}$ and $a_i = \frac{1}{i\nu}$. with

$$m_t = \rho_m m_{t-1} + \varepsilon_t^m \tag{36}$$

consist of LM curve.

As mentioned above, LM curve can be replaced with the following linearized Taylor rule

$$i_t = \phi_y\left(y_t - y_n\right) + \phi_\pi \pi_t + v_t$$

7.2 Aggregate supply block

Now we introduce the log-linearized aggregate supply block of the model under sticky price. Since labor supply at the steady state is $N_{ss}^{\varphi+1} = Y_{ss}^{-\sigma} W_{ss}/P_{ss}$, the log-deviation of labor supply (25) from its steady state value is

$$N_{ss}^{\varphi}\left[1 + \varphi n_t\right] = Y_{ss}^{-\sigma} W_{ss}/P_{ss}(1 - \sigma y_t)(1 + w_t)(1 - p_t)$$

Then,

$$\varphi n_t + \sigma y_t = w_t - p_t$$

The production function is

$$Y_{ss}(1 + y_t) = A_{ss}(1 - a_t) N_{ss}^{1-\alpha}\left[1 + (1 - \alpha) n_t\right].$$

Thus

$$y_t = a_t + (1-\alpha)n_t$$

where a_t is a log-deviation of A_t from its steady state value, and

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

7.2.1 Aggregate labor demand

To derive expression for aggregate price setting behavior or aggregate labor demand, first we derive the expression for an individual firm's and economy's average real marginal cost. First, the economy's average real marginal cost is defined

$$MC_t \equiv \frac{\partial TC_t/\partial Y_t}{P_t} = \frac{1}{P_t}\frac{\partial TC_t/\partial N_t}{\partial Y_t/\partial N_t} = \frac{W_t/P_t}{MPN_t}$$

and $MPN_t = (1-\alpha)A_t N_t^{-\alpha}$. Then, the log-deviation of the economy's average real marginal cost from its steady state is

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (w_t - p_t) - (a_t - \alpha n_t) \\ &= (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) \end{aligned}$$

for all t, where the second equality use the log deviation of output

$$y_t = a_t + (1-\alpha)n_t$$

and

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Labor supply is given by

$$\begin{aligned} w_t - p_t &= \varphi n_t + \sigma n_t \\ &= -\frac{1}{1-\alpha}a_t + \frac{\varphi + \sigma(1-\alpha)}{1-\alpha}y_t \end{aligned}$$

The equilibrium in labor market implies

$$mc_t = -\frac{1+\varphi}{1-\alpha}a_t + \frac{\alpha + \varphi + \sigma(1-\alpha)}{1-\alpha}y_t \tag{37}$$

Then, substituting (37) into (5.2) yields

$$\pi_t = \beta E_t \pi_t + \kappa\left[\frac{\alpha + \varphi + \sigma(1-\alpha)}{1-\alpha}y_t - \frac{1+\varphi}{1-\alpha}a_t\right] \tag{38}$$

Equation (38) is often referred to as the new Keynesian Phillips curve (or NKPC, for short), and constitutes aggregate supply block of the model.

Unlike traditional Phillips curve, the new Keynesian Phillips curve implies that real marginal cost is the driving force for the inflation process. It also implies that the inflation process is forward-looking, with current inflation dependent of an expected future inflation. When a firm sets its price, it must be concerned with inflation in the future because it may be unable to adjust its price for several periods.Another major difference of the new Keynesian Phillips curve from traditional Phillips curves is that the former have been derived explicitly from a model of optimizing behavior on the part of price setters, conditional on the assumed economic environment (monopolistic competition, constant elasticity demand curves, and randomly arriving opportunities to adjust prices). One advantage such a derivation provides is that it reveals how λ, the impact of real marginal cost on inflation, depends on the structural parameters β and θ. An increase in β means that the firm gives more weight to future expected profits. As a consequence, λ. declines; inflation is less sensitive to current marginal costs. Increased price rigidity (a rise in θ) reduces λ; with opportunities to adjust arriving less frequently, the firm places less weight on current marginal cost (and more on expected future marginal costs) when it does adjust its price.

7.3 Equilibrium dynamics

Following convention, we define the deviation of output gap as $\tilde{y}_t = y_t - y_t^n$, where

$$y_t^n = \Psi a_t \tag{39}$$

is the log deviation of natural level of output from its steady state with $\Psi = \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi}$. By combining (39) with (38) one can obtain an equation relating inflation to its one period ahead forecast and the output gap:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \tilde{y}_t \tag{40}$$

where $\lambda \equiv \kappa\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)$.

The second key equation describing the equilibrium of the new Keynasian model can be obtained by rewriting (IS curve) as

$$y_t = E_t\{y_{t+1}\} - \sigma^{-1}\left[i_t - E_t\{\pi_{t+1}\}\right]$$

and in terms of the output gap as follows

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \sigma^{-1}\left[i_t - E_t\{\pi_{t+1}\} - r_t^n\right] \tag{41}$$

where r_t^n is the natural rate of interest, given by

$$\begin{aligned} r_t^n &= \sigma E_t\{\triangle y_{t+1}^n\} \\ &= \sigma \Psi E_t\{\triangle a_{t+1}\} \\ &= \sigma \Psi (\rho_a - 1) a_t \end{aligned} \tag{42}$$

where the fact that $E_t\{\triangle a_{t+1}\} = E_t\{(\rho_a - 1)a_t + \varepsilon^{\alpha}_{t+1}\} = (\rho_a - 1)a_t$ is used.

Equation (41) as the dynamic IS equation (or DIS, for short). Under the assumption that the effects of nominal rigidities vanish asymptotically, we will have $lim_{T\to\infty} E_t\{\tilde{y}_{t+T}\} = 0$. In that case one can solve equation (41) forward to yield the expression

$$\begin{aligned}\tilde{y}_t &= -\,\sigma^{-1}\sum_{k=0}^{\infty} E_t\left\{r_{t+k} - r^n_{t+k}\right\} \\ &= -\,\sigma^{-1}\sum_{k=0}^{\infty} E_t\left\{i_{t+k} - \pi_{t+k+1} - r^n_{t+k}\right\}\end{aligned}$$

which implies that the output gap depends on the future gaps between the actual and the natural interest rate. Since the long-term interest rate can be defined as the sum of the short-term interest rates, another way of seeing the IS is that the output gap depends on the difference between the long-term interest rate and the natural long-term interest rate. Iterating (40) forward:

$$\pi_t = E_t\{\beta^k + \lambda\tilde{y}_{t+k}\} \tag{43}$$

Equations (40) and (41), together with an equilibrium process for the natural rate r^n_t (which in general will depend on all the real exogenous forces in the model), constitute the non-policy block of the basic new Keynesian model. That block has a simple recursive structure: the NKPC determines inflation given a path for the output gap, whereas the DIS equation determines the output gap given a path for the (exogenous) natural rate and the actual real rate. The equations appear broadly similar, however, to the types of aggregate demand and aggregate supply equations commonly found in intermediate-level macroeconomics textbooks. However, both equations are derived from well-specified optimization problems, with (41) based on the Euler condition for the representative agent's decision problem and (40) derived from the pricing decisions of individual firms.

Equations (40) and (41) contain three variables: the output gap, inflation, and the nominal interest rate. In order to close the model, we need to supplement those two equations with one or more equations determining how the nominal interest rate it evolves over time, i.e. the money market equilibrium condition, the LM curve:

$$m_t - p_t = a_c\tilde{y}_t - a_i i_t - a_c\Psi a_t$$

where m_t is a money supply given by (36).

8 Equilibrium Dynamics under Interest Rate Rule

If the central bank picks the nominal interest rate as the monetary policy instrument, as opposed to a money supply aggregate, then it is not necessary to specify a money market equilibrium. With the nominal interest rate as the instrument of monetary policy, the central bank adjusts the money supply to hit the interest rate target. The condition that money demand equals money supply simply determines the value of the money supply that meets this criteria. In this case, the LM curve determines what the accommodating money stock will be, and nothing else. Thus, the LM curve is not relevant and we can ignore it. Now we assume that the central bank use the interest rate as the instrument, and analyze the equilibrium under a simple interest rate rule of the form:

$$i_t = \phi_y \tilde{y}_t + \phi_\pi \pi_t + v_t \tag{44}$$

where v_t is an exogenous (possibly stochastic) component with zero mean, and it is assumed that the exogenous component of the interest rate v_t follows an AR(1) process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

where $\rho \in [0, 1)$. It is also assumed that ϕ_y and ϕ_π are non-negative coefficients, chosen by the monetary authority. Equations (40) and (41), together with an equilibrium process for the natural rate r_t^n,and (44) represent the equilibrium conditions of the basic New Keynesian model.

Substituting (42) and (44) into (40) and (41) yields

$$\begin{aligned}
\pi_t &= \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \\
\tilde{y}_t &= E_t\{\tilde{y}_{t+1}\} - \sigma^{-1}\left[\phi_y \tilde{y}_t + \phi_\pi \pi_t + v_t - E_t\{\pi_{t+1}\} - \sigma\Psi(\rho_a - 1)a_t\right]
\end{aligned}$$

Next, guess that the solution takes the form

$$\tilde{y}_t = \Lambda_{yv} v_t + \Lambda_{ya} a_t \tag{45}$$

$$\pi_t = \Lambda_{\pi v} v_t + \Lambda_{\pi a} a_t \tag{46}$$

where Λ_{ij} are coefficients to be determined. Imposing the guessed solution on (45) and (46), and using the method of undetermined coefficients, Λ_{ij} can be determined.

Next the economy's equilibrium response to two exogenous shocks—a monetary policy shock and a technology shock—is examined when the central bank follows the interest rate rule (44).

8.1 The Effects of a Monetary Policy Shock

Setting $a_t = 0$, for all t for the purposes of this exercise(i.e., turning off technological shocks). Since the natural rate of interest is not affected by a monetary policy shock, $r_t^n = 0$ for all t . Then, guess that the solution takes the form

$$\tilde{y}_t = \Lambda_{yv} v_t$$

$$\pi_t = \Lambda_{\pi v} v_t$$

where Λ_{yv} and $\Lambda_{\pi v}$ are coefficients to be determined. Imposing the guessed solution on (45) and (46), and using the method of undetermined coefficients,

$$\Lambda_{yv} = -\frac{(1-\beta\rho_v)}{1-\beta\rho_v)[\sigma(1-\rho_v)+\phi_y]+\kappa(\phi_\pi-\rho_v)}$$

and

$$\Lambda_{\pi v} = \frac{\kappa}{(1-\beta\rho_v)}\Lambda_{yv}$$

where $\Lambda_{yv} > 0$ as long as equilibrium exits, $\phi_\pi > 1$. Hence, an exogenous increase in the interest rate leads to a persistent decline in the output gap and inflation. Because the natural level of output is unaffected by the monetary policy shock, the response of output matches that of the output gap. The previous analysis can be used to quantify the effects of a monetary policy shock, given numerical values for the model's parameters. We will discuss this in the next chapter.

8.2 The Effects of a Technology Shock

In order to determine the economy's response to a technology shock first a process must be specified for the technology parameter a_t and then an implied process can be derived for the natural rate. Setting $v_t = 0$, for all t (i.e., turning off monetary shocks), and guessing that output gap and inflation are proportional to a_t, the method of undetermined coefficients can be applied in a way analogous to the previous subsection to obtain

$$\tilde{y}_t = \Lambda_{ya}a_t$$
$$\pi_t = \kappa\Lambda_{ya}a_t$$

where $\Lambda_{ya} = -\sigma\Psi(1-\rho_a)\frac{(1-\beta\rho_a)}{1-\beta\rho_a)[\sigma(1-\rho_a)+\phi_y]+\kappa(\phi_\pi-\rho_a)} > 0$. Hence, and as long as $\rho_a < 1$, a positive technology shock leads to a persistent decline in both inflation and the output gap. Numerically analysis of a technological shock will be discussed in the next chapter.

Appendix: Log-Linearization of Money demand

We log-linearize the money demand function. This is little bit tricky. We can write money demand function of (21):

$$\frac{M_t}{P_t} = \left(\frac{1+i_t}{i_t}C_t^\sigma\right)^{\frac{1}{\nu}}. \tag{47}$$

Write the (47) as:

$$\ln M_t - \ln P_t = \frac{1}{\nu}\left\{\ln\frac{1+i_t}{i_t} + \sigma\ln C_t\right\}. \tag{48}$$

However,

$$\ln\frac{1+i_t}{i_t} = \ln\left(\frac{1+i_t-1}{1+i_t}\right)^{-1} = -\ln\left(1-\frac{1}{1+i_t}\right) = -\ln\left(1-\frac{1}{e^{\ln(1+i_t)}}\right). \tag{49}$$

Using $\ln(1+i_t) \approx i_t$ for i_t close to zero and (49), we can write (48) as:

$$m_t - p_t = \frac{1}{\nu}\left\{\ln\left(1 - e^{-i_t}\right) + \sigma \ln C_t\right\}.$$

Take the first-order Taylor expansion of $\ln(1 - e^{-i_t})$ around steady state $i_t = i$ up to constants

$$\begin{aligned}
\ln(1 - e^{-i_t}) &\approx \ln(1 - e^{-i}) + \frac{e^{-i}}{1 - e^{-i}}\left[i_t - i\right] \\
&\approx \ln(1 - e^{-i}) - \frac{1}{1 - e^{-i}}(ie^{-i}) + \frac{1}{1 - e^{-i}}(e^{-i}i_t) \\
&\approx costant + \frac{1}{e^{i} - 1} i_t \qquad (50)
\end{aligned}$$

Using $e^i \approx e^{\ln(1+i)} \approx 1 + i$ and (50), we can write as

$$\ln M_t - \ln P_t = \frac{1}{\nu}\left(constant - \frac{1}{i} i_t + \sigma \ln C_t\right)$$

Then the log-deviation of money demand from the steady-state is

$$m_t - p_t = a_c c_t - a_i i_t$$

where $a_c \equiv \frac{\sigma}{\nu}$ and $a_i = \frac{1}{\nu}\frac{1}{i}$.

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