Project: Risk Analysis Using Econometric Models

Due on December 10, 2020

- **Aim.** This project helps you practice all techniques taught in this course by solving questions in risk management. Some specific techniques include: exploring data, fitting parametric distribution family to univariate data, fitting copula models, fitting AR-GARCH models, diagnosing time series models, computing risk measures, etc.

- **Datasets.** Use R to download the recent ten years stock prices for SPXL and one bank or company chosen by you from Yahoo Finance.

- **Explore Data.** Explore both data sets to argue that it is necessary to model time dependence and volatility of log-returns.

- **Build Time Series Models.** First fit an AR(1)-GARCH(1,1) model to each series of log-returns:

$$X_t = \mu + \phi X_{t-1} + e_t,\ e_t = \sigma_t \varepsilon_t,\ \sigma_t^2 = w + \alpha e_{t-1}^2 + \beta \sigma_{t-1}^2,\ t = 1, \cdots, n$$

and

$$\tilde{X}_t = \tilde{\mu} + \tilde{\phi} \tilde{X}_{t-1} + \tilde{e}_t,\ \tilde{e}_t = \tilde{\sigma}_t \tilde{\varepsilon}_t,\ \tilde{\sigma}_t^2 = \tilde{w} + \tilde{\alpha} \tilde{e}_{t-1}^2 + \tilde{\beta} \tilde{\sigma}_{t-1}^2,\ t = 1, \cdots, n.$$

Second fit parametric distribution families to both ε_t and $\tilde{\varepsilon}_t$ (say, standardized t-distributions in the sense of having variance one), and a parametric copula family to model the copula of ε_t and $\tilde{\varepsilon}_t$ (say t-copula). Therefore, we have a parametric model for $(\varepsilon_t, \tilde{\varepsilon}_t)$ denoted by $F_1(x; \boldsymbol{\theta}_1), F_2(x; \boldsymbol{\theta}_2), C(x_1, x_2; \boldsymbol{\theta}_3)$. For all fittings, you should diagnose the fittings and argue they are reasonable.

- **Risk Calculation.** To forecast the Value-at-Risk at level 99% of $X_{n+1} + \tilde{X}_{n+1}$ given $\mathcal{F}_n$, the σ-field generated by $X_1, \cdots, X_n, \tilde{X}_1, \cdots, \tilde{X}_n$, we solve the following equation to x:

$$0.99 = P(X_{n+1} + \tilde{X}_{n+1} \leq x | \mathcal{F}_n) = P(\mu + \phi X_n + \sigma_{n+1}\varepsilon_{n+1} + \tilde{\mu} + \tilde{\phi}\tilde{X}_n + \tilde{\sigma}_{n+1}\tilde{\varepsilon}_{n+1} \leq x | \mathcal{F}_n).$$

  Note that, given $\mathcal{F}_n$, $\mu, \phi, X_n, \sigma_{n+1}, \tilde{\mu}, \tilde{\phi}, \tilde{X}_n, \tilde{\sigma}_{n+1}$ are constants and available in the model fitting above. So to compute the above probability, we could employ simulation method: draw a random sample with a big sample size B from copula $C(u_1, u_2; \boldsymbol{\theta}_3)$, say $\{(U_i, V_i)\}_{i=1}^B$; make transformations $\varepsilon_i^* = F_1^-(U_i; \boldsymbol{\theta}_1)$ and $\tilde{\varepsilon}_i^* = F_2^-(V_i; \boldsymbol{\theta}_2)$ for $i = 1, \cdots, B$, where F_i^- means the inverse function of F_i. Therefore estimating the Value-at-Risk is to solve the following equation to x:

$$0.99 = \frac{1}{B} \sum_{i=1}^{B} I(\mu + \phi X_n + \sigma_{n+1}\varepsilon_i^* + \tilde{\mu} + \tilde{\phi}\tilde{X}_n + \tilde{\sigma}_{n+1}\tilde{\varepsilon}_i^* \leq x),$$

  where $I(x)$ denotes the indicator function. Similarly, compute the expected shortfall at level 99%.

- **Report.** Your report should include an introduction section on datasets and aims, a section on the statistical analysis described above, and a conclusion section on what your findings are. It would be good to add your thought on this course to the conclusion section. For example, what have you mastered after this class? How is the balance between theory and application? What do you like to know more?