Assignment 7

1. Create simple matrices (preferably at most 3x4) and show that
   1. Ax(B+C) = (AxB)+(AxC)
   2. $(A+B)^T = A^T + B^T$
   3. $(AxB)^T = B^T \times A^T$
2. Show that the determinant of a diagonal matrix is the product of the diagonal elements
3. Find the "inverse" of your diagonal matrix (same diagonal matrix from question 2)
4. Generate the variance-covariance matrix (S) for the USArrets data using Centering and Indempotent matrices (see hint on the next slide) and compare your result with the built-in R function for calculating the variance-covariance matrix
5. Calculate the "trace" of this variance-covariance matrix. What does this result mean/show?
6. Calculate the "determinant" this variance-covariance matrix. What does this result mean/show?
7. Compare the results of "trace" and "determinant" calculations. Which one has a larger value? Why?
8. Find the eigen values and eigen vectors for the variance-covariance matrix of the USArrets data
   - Comment on your findings. Such as, how many positive eigen values, how many zero eigen values, what does this tell you about the rank of the variance-covariance matrix ?

Hint:

$$
\begin{aligned}
\boldsymbol{S} &= \frac{1}{n-1}\sum_{i=1}^{n}(\boldsymbol{x}_i - \bar{\boldsymbol{x}})^T(\boldsymbol{x}_i - \bar{\boldsymbol{x}}) \\
&= \sum_{i=1}^{n}\boldsymbol{x}_i\sum_{i=1}^{n}\boldsymbol{x}_i^T - \bar{\boldsymbol{x}}\bar{\boldsymbol{x}}^T \\
&= \frac{1}{n-1}\boldsymbol{X}^T\boldsymbol{X} - \bar{\boldsymbol{x}}\bar{\boldsymbol{x}}^T \\
&= \frac{1}{n-1}\left(\boldsymbol{X}^T\boldsymbol{X} - \frac{1}{n}\boldsymbol{X}^T\mathbf{1}\mathbf{1}^T\boldsymbol{X}\right)
\end{aligned}
$$

$$
\boldsymbol{S} = \frac{1}{n-1}\boldsymbol{X}^T\boldsymbol{H}\boldsymbol{X}
$$