**MAT 364 Project 3 – Benchmark - Mass-Springs, Circuits, and Resonance**

The goal of this project is to construct a simple mass-spring mathematical model, expressed as a first order system of differential equations, representing the suspension and dampener for a particular motor vehicle, and then investigate how solutions of such a system behave. Furthermore, you are to also investigate how certain external forces cause such physical systems to react. You will be responsible for constructing and designing your mathematical models under the conditions provided below.

Part 1: Design and Initial Solutions

A) First, identify the type of vehicle you will be using to construct a mathematical model based upon your student ID number. Let *A* be the last digit of your student ID number.

Choose and state a vehicle from the assigned class above. Next, to construct your mathematical model, you may assume that the mass of the vehicle is distributed evenly throughout the car (for simplicity). Here, *m* (in kg) will represent the mass of the vehicle, the dampener attached to the suspension will have a damping coefficient of *c* (in N/(m/s)), and the spring constant, otherwise known as the spring rate, of the coil in the suspension application will be *k* (in N/m). Without an external force acting on the suspension, the second order differential equation governing this system is given below.

Enter in your value for the mass, converted into kg. To determine the spring rate, you may either research known values of *k* for suspension coils for your specific vehicle (cite sources if you take this route), or, you can calculate this value yourself. For this second route, use the given equation on the following page relating different aspects of the material and structure of a suspension coil appropriate for your vehicle.

Note that *d* is the diameter of the wire of the coil, *G* is the spring’s shear modulus, *N* is the number of full wraps of the coil, and *D* is the diameter of the coil itself. A valuable reference to find information on suspension coils is given below.

<https://www.moog-suspension-parts.com>

<https://www.moog-suspension-parts.com/universal_coil_springs.asp>

B) Write this second order differential equation as a first order matrix-vector system of differential equations. Show your work in detail.

Now, suppose the front left portion of your vehicle drifts off a curb of height 0.15 meters without an initial velocity. Write an appropriate initial condition using proper notation describing this event and pair it with your first order system.

C) Use the IVP Solver script in MATLAB to plot solutions to your system with small, varying choices of the damping coefficient *c*. Use the hold on command to plot many different solution curves on the same set of axes. Provide your plot and code in your final submission. Leave comments noting what values of *c* were used in the script.

Now, select an appropriate choice for *c*, based on your results above, that causes no more than two oscillations of the vehicle. This will be your damping coefficient for the remainder of the project.

D) Create a phase portrait of your first order system using the Phase Portrait script in MATLAB. Provide your plot and code in your final submission. Classify the type and stability of the critical point at the origin, then describe why your phase portrait matches your solution determined using the IVP Solver.

Part 2: External Forces, Resonance

A) Return to your second order differential equation with your values for *m*, *c*, and *k* entered into it, and note the initial conditions stated earlier. Solve this differential equation using any appropriate method discussed in class.

B) Based on the solution to your IVP above, identify a function that if acting on the system as an external force (like your vehicle driving over a bumpy road) would cause mechanical resonance to occur in the system. Justify why this choice would cause resonance to occur.

C) Set up a slightly modified second order differential equation of the form below.

Here, the function on the right-hand side of the equation should be the external force identified at the end of the prior page. This system should experience mechanical resonance if the choice made above is correct.

Write this second order differential equation as an inhomogeneous first order matrix-vector system of differential equations. Show your work in detail. Next, use the IVP Solver script in MATLAB to plot the solution to this differential equation. Provide your plot and code in your final submission. Describe the changes in the solutions for your system with and without the presence of the external force.

Part 3: Relationship with Circuits

A) Virtually every portion of the results above can be duplicated in the setting for a closed circuit with an external voltage. Recall the second order differential equation for a closed RLC circuit, given here.

Determine the values of *L*, *R*, *C*, the function, *e*(*t*), and the initial conditions *q*0 and *i*0 that would create an exact duplicate of the mass-spring work above with respect to circuits. Ensure that proper and correct units are included in your results throughout.

B) If a circuit undergoes resonance, describe what this means in context. Lastly, further explain the differences and similarities between resonance occurring in a mass-spring system versus resonance occurring in a closed circuit, and take particular care in describing the phenomena scientifically.