Problem 9: X-ray Flares

Assigned: 24 November
Due: 18 December (5:00 PM)
Maximum Mark: 15 Points

You are analyzing an object that normally does not produce X-rays, but every once in a while it flares up, emits a huge number (S) of X-rays randomly in all directions, and then fades away. Each flare is thought to be basically identical. We have placed a detector near the source, which is expected to intercept one out of every thousand X-ray photons *on average*. However, the actual number of X-ray photons received during each flare may differ due to counting statistics.

1. After a long wait, the source finally flares and we count a total of n = 5 X-ray photons in our detector. Based on this measurement, and knowing that the detector intercepts 1 in 1000 photons on average, estimate the value of S and provide a (non-approximate) 95% frequentist confidence interval.

2. Write down the equation for the likelihood function L(S; n) for this scenario (for arbitrary values of S and n). Then, make a (1D) plot of the likelihood L as a function of S given our data (n=5).

3. What is the maximum-likelihood value of S, if n=5? (You can calculate numerically based on the plotted curve in #2, or prove an exact result analytically if you prefer.)

4. What is an approximate 95% credible interval on S, if n=5? (Assume flat priors. You can estimate the limits of the interval numerically.)

5. We wait a long time, and the source flares again. During this new episode, we count n=2 photons. Based on this new information (and the information already available to us), calculate a new best estimate for S, a new frequentist confidence interval, and a new Bayesian credible interval.