MATH 447/647: Probability Models
Fall 2020 Semester
Homework 7: Due Monday December 7, 5:30pm ET.

Instructions

- Write your full name, "Homework 7", and the date at the top of the first page.
- Show all work, including each step of your solution, to earn maximal partial credit.
- Each question has multiple parts. Write legibly and neatly. Box your final answers.
- Use Genius Scan or a similar application to convert your solutions to .pdf format.
- Submit a single .pdf file to Gradescope under the assignment "Homework 7".
- If you have any questions, email me or come to office hours (WF 11:00am-12:00pm)
- You are encouraged to work together (on Piazza) but must write up your own solutions.

Assignment (3 Problems: 60 + 20 + 20 = 100 points total.)

☐ **Problem 1** The evening of October 31st, my doorbell rang more often than it usually does. Each time I opened the door, total strangers were there expecting me to give them sweet delicacies for free. I stopped answering the door but the doorbell kept ringing all night at an average rate of 2 rings per minute. I gave up on getting sleep and tried to predict when the next bell would ring by modeling the rings as a Poisson process.

- 1.1 [10 points] What is the probability there are 3 rings between 8:00pm and 8:02pm?
- 1.2 [10 points] What is expected number of rings between 8:00pm and 8:02pm?
- 1.3 [10 points] Given that the bell rang at 9:30pm, what is the probability that the next ring occurs exactly at 9:34pm?
- 1.4 [10 points] Given that the bell rang at 9:30pm, what is the probability that the next ring occurs after 9:34pm?
- 1.5 [10 points] Given that the bell did not ring at all between 9:30pm and 9:34pm, what is the probability that the next ring occurs after 9:37pm?
- 1.6 [10 points] Given that the bell rang 5 times between 9:30pm and 9:34pm, what is the probability that the bell rings 5 times between 10:00pm and 10:02pm?

☐ **Problem 2** [20 points] Suppose N is a positive integer and $\gamma > 0$ a positive real number. For fixed k, calculate the quantity $C_\gamma(k)$ defined by

$$C_\gamma(k) := \lim_{N\to\infty} \frac{N!}{k!(N-k)!}\left(\frac{\gamma}{N}\right)^k \left(1-\frac{\gamma}{N}\right)^{N-k}.$$

☐ **Problem 3** [20 points] Let Y_1 and Y_2 be independent exponential random variables with respective rates λ_1 and λ_2. Calculate the probability density function of $T = \min(Y_1, Y_2)$.

☐ **Bonus** [X points] For any random variable Y, the *moment generating function* of Y is the function of t in $\mathbb{R}$ defined by $\mathbb{E}[e^{tY}]$. Calculate the moment generating function of an exponential random variable Y with parameter $\lambda > 0$.