1. **(22 points) Consider a two-period economy where the representative firm uses a production technology given by $Y_i = z_i F(K_i, N_i)$ for $i = 1, 2$, where K_i is the capital input, that depreciates at the rate δ, and N_i is the labor input used by the firm in period i to produce Y_i.**
Suppose that in period i, for i $= 1, 2$, the government levies a proportional tax, τ_i, where $\tau_i \in (0, 1)$ on firms revenue Y_i. So the taxes paid by the representative firm are $\tau_i Y_i$. The tax revenues are rebated lump-sum to households. Let TR_i denote the lump sum transfer that the representative consumer receives.
The representative firm owns capital, pays a wage rate w per unit of labor hired and can borrow or lend at the market real interest rate r. It decides on how much labor to hire in each period and how much capital to accumulate for the following period in order to maximize the present discounted value of after-tax profits:

$$\max_{N_1, K_2, N_2} V = \Pi_1(N_1, K_2, K_1, \tau_1, z_1) + \frac{\Pi_2(N_2, K_2, \tau_2, z_2)}{1+r}$$

   (a) **4 points) Write the equations describing the profit (pre-tax and after-tax) of the representative firm in both periods.**

   (b) **(4 points) Set-up the representative firm's problem, and derive the corresponding optimality conditions.**

   (c) **(2 points) Assume that the government balances its budget in every period. Write the government's budget constraints:**

   Consider now that in this economy the representative consumer maximizes

$$U = u(C_1) + \beta u(C_2)$$

   where C_t is per capita consumption. $0 < \beta < 1$.
   The representative consumer receives disposable income in each period: $(w_1 N_1 + \Pi_1 + TR_1)$ **in period 1 and** $(w_2 N_2 + \Pi_2 + TR_2)$ **in period 2, supplies $N_1 = N_2 = 1$ units of labor in each period and can borrow or lend at the market interest rate, r. TR_i for i $= 1, 2$ denote the lump sum transfer that the representative consumer receives in each period as described previously.**
   Assume the consumer allocates resources across both periods consumption according to the optimality condition

$$u_C(C_1) = (1+r)\beta u_C(C_2)$$

(d) **(4 points) Write the system of equations that defines the competitive equilibrium for this economy. List all the variables that you are solving for and make sure you have enough equations to solve for all the variables**

Assume that the production function takes the form, $F(K, N) = K^{\alpha} N^{1-\alpha}$**, while** $\delta = 1$ **and the momentary utility takes the following functional form:**

$$u(C) = \log C.$$

(e) **(6 points) Solve for the competitive equilibrium level of capital accumulation,** K_2**.**

(f) **(6 points)How does capital accumulation respond to an increase in the tax rates,** τ_t **for** $t = 1, 2$**? How does consumption respond in each period? Explain intuitively.**

2. **(10 points) Impact of government spending: Tax on consumption with government spending**
Consider a two-period economy where the representative consumer has a lifetime utility over consumption, C_t for t=1,2, **defined as**

$$U(C_1, C_2) = \log C_1 + \beta \log C_2,$$

where $\beta > 0$. **In each period** $t=$ **1, 2, the consumer provides** $N_S = 1$ **units of labor to firms at wage rate** w_t**, receives profits,** Π_t**, from firms.**
The government levies a tax τ_t **on consumption and uses the revenue in each period to finance government expenditures,** G_t**.**

The total amount of taxes paid by the representative consumer in period t **are given by** $\tau_t C_t$**.**
The consumer can borrow or lend between period 1 and period 2 at real interest rate r **and chooses consumption** (C_1, C_2) **and savings to maximize their utility.**
The representative firm has a production technology given by

$$Y = AF(K, N) = AK^{\alpha}N^{1-\alpha}, \; 0 < \alpha < 1$$

where K **is capital input and** N **is labor input. Assume capital depreciates at rate** $\delta = 1$.
A competitive equilibrium for this economy is fully described by the following set of equations

$$\begin{cases} (1+\tau_2)\, C_2^* = (1+r_t^*)\, \beta\, (1+\tau_2)\, C_1^* \\ \\ 1 + r = z_2 F_K\left(K_2^*, N_2^*\right) + 1 - \delta \\ \\ I_1^* = K_2^* - (1-\delta)\, K_1 \\ \\ I_2^* = -(1-\delta)\, K_2^* \\ (1+\tau_t)\, C_t^* = w_t^* N_S + \Pi_t^* \text{ for t=1,2} \\ \\ w_t^* = z_t F_N\left(K_t^*, N_t^*\right) \text{ for t=1,2} \\ \\ Y_t^* = AK_t^{*\alpha}\left(N_t^*\right)^{1-\alpha} \text{ for t=1,2} \\ \\ \Pi_t^* = Y_t^* - w_t^* N_t^* - I_t^* \text{ for t=1,2} \\ \\ G_t^* = \tau_t C_t^* \text{ for t=1,2} \\ \\ C_t^* + G_t^* + I_t^* = Y_t^* \text{ for t=1,2} \\ \\ N_s = N_t^* = 1 \end{cases}$$

(a) **(6 points) Solve for the competitive equilibrium level of capital in the second period.**

(b) **(4 points)How does capital accumulation respond to the tax rate on consumption, τ_t for $t = 1, 2$? Explain intuitively (difficult, see second midterm).**