Assignment 3 : MA3140

Q 1. Let $X_1, \ldots, X_n$ be a r.s. from Pareto distribution with density $f(x) = \frac{\beta\alpha^\beta}{x^{\beta+1}}$, $x > \alpha$, $\alpha > 0$, $\beta > 2$. Find a sufficient statistic when (i) α is known; (ii) β is known; and (iii) both α and β are unknown.

Q 2. Let $X_1, \ldots, X_n$ be a r.s. from $Beta(\lambda, \mu)$ population. Find a sufficient statistic when (i) μ is known; (ii) λ is known; and (iii) both μ and λ are unknown.

Q 3. Let $X_1, \ldots, X_n$ be a r.s. from a continuous population with density function $f(x) = \frac{\theta}{(1+x)^{1+\theta}},\ x > 0,\ \theta > 0$. Find a minimal sufficient statistic.

Q 4. Let $X_1, \ldots, X_n$ be a r.s. from a Geometric population with pmf $f(x) = p(1-p)^{x-1},\ x = 1, 2, \ldots,\ 0 < p < 1$. Find a minimal sufficient statistic.

Q 5. Let $X_1, \ldots, X_n$ be a r.s. from an exponential population with density function $f(x) = e^{\mu - x},\ x > \mu,\ \mu \in \mathbb{R}$. Show that $Y = X_{(1)}$ is complete.

Q 6. Let $X_1, \ldots, X_n$ be a r.s. from a $N(\theta, \theta^2)$ population. Show that $(\overline{X}, S^2)$ is minimal sufficient but not complete.

Q 7. Let $X_1, \ldots, X_n$ be a r.s. from a $N(0, \sigma^2)$ population. Show that X is not complete but X^2 is complete.

Q 8. Let $X_1, \ldots, X_n$ be a r.s. from a $P(\lambda)$ population. Find a UMVUE of $g(\lambda) = P(X_1 \leq 1) = (1+\lambda)e^{-\lambda}$.

Q 9. Let $X_1, \ldots, X_n$ be a r.s. from an exponential population with density function $f(x) = e^{\mu - x},\ x > \mu,\ \mu \in \mathbb{R}$. Find UMVUEs of μ and μ^2.

Q 10. Let $X_1, \ldots, X_n$ be a r.s. from a $N(\mu, \sigma^2)$ population. Find UMVUEs of the signal to noise ratio μ/σ and quantile $\mu + b\sigma$, where b is any real number.