MATH 5640 Homework 7 (due on Thursday 11/19/2020)

(You must show details of your work in order to get full credits)

1. Let $u = (1, 2, 2, 4)$. Use an ***elementary projector*** to find an orthonormal basis in $\mathbb{R}^4$ that contains the unit vector $\frac{u}{\|u\|}$.

2. Suppose $S \in \mathbb{R}^{n\times n}$ and $T \in \mathbb{R}^{n\times n}$ are two elementary reflectors that are not identical. Under which condition do we have $ST = TS$?

Hint: Wrire $S = I - 2aa^T$ and $T = I = 2bb^T$ for some unit vectors $a, b \in \mathbb{R}^n$. Then find conditions for a, b such that $ST = TS$.

3. Let m, n be positive integers. Let $R \in \mathbb{R}^{n\times n}$ be an elementary reflector.

Is $S = \begin{pmatrix} I_{m\times m} & 0 \\ 0 & R \end{pmatrix} \in \mathbb{R}^{(m+n)\times(m+n)}$ an elementary reflector?

4. Let $A \in \mathbb{R}^{m\times n}$ with rank $r = \text{rank}(A)$. Suppose $P \in \mathbb{R}^{m\times m}$ is an orthogonal matrix such that

$$P^T A = \left(\begin{array}{c|c} A_1 & A_2 \\ \hline 0 & 0 \end{array}\right)$$

where $A_1 \in \mathbb{R}^{r\times r}$ is upper triangular with nonzero diagonal entries and $A_2 \in \mathbb{R}^{r\times n}$.

Show that the first r columns of P is an orthonormal basis for $R(A)$ and the last $m - r$ columns of P is an orthonormal basis for $N(A^T)$.

Hint: left-multiply both sides by P, then express columns of A as linear combinations of columns of P, and use the orthogonal complement $R(A)^\perp = N(A^T)$.

5. Let

$$A = \begin{pmatrix} 2 & 0 & 3 & 1 \\ 4 & 2 & 0 & 2 \\ -4 & -1 & -3 & -2 \end{pmatrix}.$$

Use Problem 4 and Householder reduction (see the lecture note *Orthogonal Reduction*) to obtain orthonormal bases for $R(A)$ and $N(A^T)$. You may use software for matrix computations but the function `qr()` in Matlab is not allowed.

Practice Problems

(For your practice only! Do not submit these problems as they will not be graded)

1. Let

$$L = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 0 & -1 \\ 1 & 1 & 1 \end{pmatrix}$$

Use the Gram-Schmidt procedure to obtain a QR factorization for L (You may use software for the computations).

2. Use Householder reduction to find a QR factorization of L (You may use software for the computations).

3. Use Givens reduction to find a QR factorization of L (You may use software for the computations).

4. Let

$$A = \begin{pmatrix} 2 & 0 & 3 & 1 \\ 4 & 2 & 0 & 2 \\ -4 & -1 & -3 & -2 \end{pmatrix}.$$

Similar to problem **5** above, obtain orthonormal bases for $R(A^T)$ and $N(A)$. Then use the results to construct a URV decomposition for A. (You may use software for the computations).