

IMPORTANT: Homework must be typeset. (Why? The homework has to be collected electronically. Grading handwritten homework was difficult. Grading photographs of handwritten homework is prohibitively difficult.)

Homework 4 due: Nov. 19 (Thursday), 11am; submit on Gradescope.

Problem sessions before Homework 4 is due:

Nov. 16 (Monday) 6:15pm - 7:15pm in Hoyt Hall Room 104 (Akira Sah)

Nov. 17 (Tuesday) 6:15pm - 7:15pm in Hoyt Hall Room 104 (Mandar Juvekar)

Nov. 18 (Wednesday) 6:15pm - 7:15pm ONLINE (Ben Kuehnert)

Nov. 18 (Wednesday) 7:40pm - 8:40pm ONLINE (Conor Taliancich)

1 Homework - solve and turn in

4.1 (due Nov. 19) The United Coal Company owns n coal mines and m coal power plants. You have the following information:

- For each $i \in [n]$ you have:
 - C_i = the cost (in USD) to mine a metric ton of coal in the i -th mine,
 - L_i = maximum amount (in metric tons) of coal that can be mined in the i -th mine in a day.
- For each $i \in [n]$ and each $j \in [m]$ you have:
 - $T_{i,j}$ = the cost (in USD) to transport a metric ton of coal from the i -th mine to the j -th power plant.
- For each $j \in [m]$ you have
 - P_j = amount of electricity (in MWh) produced by burning a metric ton of coal in the j -th power plant (each plant has different efficiency);
 - M_j = amount of mercury (in grams) released by burning a metric ton of coal in the j -th power plant (each plant has a different pollution control system).
- Q = price of electricity (in USD per MWh).
- W = EPA limit on mercury emissions (in grams per MWh).

You want to make as much money as possible. You need to comply with the EPA limit—that means if United Coal Company (aggregated over all power plants it owns) produces x MWh of electricity per day then United Coal Company can release (aggregated over all power plants it owns) at most $W \cdot x$ grams of mercury per day.

Write a linear program that determines how much profit you can make each day (profit = price of electricity produced minus costs (in this problem we have mining costs and transportation costs)). Clearly state what are the variables in your linear program and what is their meaning.

4.2 (due Nov. 19) This problem involves three chemicals A, B, C . You have a liters of chemical A , b liters of chemical B , and c liters of chemical C (for each chemical you have a pure solution). There are n potential customers. The i -th customer wants d_i liters of an $a_i : b_i : c_i$ mixture of chemicals A, B, C and is willing to pay p_i dollars per liter of the desired mixture. You need cash. You want to get as much money as possible by creating mixtures and selling them to the potential customers. The customers will purchase your mixture even if you only partially satisfy their demand (for example, if a customer wants 3 liters of a 7 : 8 : 9 mixture and is willing to pay \$8 per liter and you supply 1.2 liters of a 7 : 8 : 9 mixture they will pay you \$9.6). Give a linear program for the problem. **Clearly describe the variables in your linear program.**

2 CSC 482 Homework - solve and turn in (bonus for CSC 282)

4.3 (do not turn in—this problem is to help with the problems below) Let a be a constant and x be a variable. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{array}{ll}\min & x \\ & x \geq a \\ & x \geq -a\end{array}$$

Let a, b be constants and x, y be variables. What is the value of the following linear program? Give a simple mathematical expression.

$$\begin{array}{ll}\min & x + y \\ & x \geq a \\ & x \geq -a \\ & y \geq b \\ & y \geq -b\end{array}$$

(If you don't know where to start—plug in concrete values for a and b (make sure to use negative numbers too), solve the resulting linear program (either by hand or using a solver) and formulate a conjecture based on the experiments.)

4.4 (due Nov. 19)

PART A:

We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\max_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

PART B:

$(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} |ax_i + y_i - b|.$$

Write a linear program for this problem.

PART C:

We are given n pairs of real numbers $(x_1, y_1), \dots, (x_n, y_n)$. We want to find real numbers a, b which minimize

$$\sum_{i \in \{1, \dots, n\}} (ax_i + y_i - b)^2.$$

Find a formula for a and b (you do NOT need linear programming for this one).