

FINAL EXAM

All problem parts have equal weight. In budgeting your time expect that some problems will take longer than others.

Remember, answers without proper justification will not receive full credit!

1. Assume that X_1, \dots, X_n are i.i.d. $\text{Gamma}(3, \lambda)$, where $\lambda > 0$.
 - (a) Find a one-dimensional sufficient statistics.
 - (b) Find the MM estimator.
 - (c) Find the MLE.
 - (d) Is the MLE estimator you derived unbiased? What is its MSE?
 - (e) Assuming that the prior is $\lambda \sim \Gamma(a, b)$. Find the posterior and the posterior Bayes estimator.
 - (f) Find the CRLB for estimating λ .

2. The following are results of independent repetitions of an experiment.

(Success is denoted by 1 and failure by 0.)

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Test $\mathcal{H}_0 : p = 0.7$ against $\mathcal{H}_1 : p > 0.7$ at $\alpha = 0.05$.

3. Let X_1, \dots, X_n be i.i.d. $N(\mu_x, \sigma_x^2)$ and Y_1, \dots, Y_m be i.i.d. $N(\mu_y, \sigma_y^2)$. The \mathbf{X} s are independent of \mathbf{Y} s.
- (a) Find the GLRT for testing $\mathcal{H}_0 : \mu_x = \mu_y, \sigma_x^2 = \sigma_y^2$ at $\alpha = 0.01$. Assume that n, m are large.
 - (b) We have observed $n = 121$, $\bar{X} = 0.065$, $S_x^2 = 0.806$ and $m = 101$, $\bar{Y} = -0.176$, $S_y^2 = 9.689$. Does the GLRT test you derived in part 3a reject the null hypothesis?
 - (c) Using the same data set as in part 3b test $\mathcal{H}_0 : \sigma_x^2 = \sigma_y^2$ vs. $\mathcal{H}_a : \sigma_x^2 \neq \sigma_y^2$ at $\alpha = 0.05$.
 - (d) Using the same data set as in part 3b find the 99% confidence interval for μ_y .

4. An experiment was conducted to compare the mean number of tape-worms in the stomachs of sheep that had been treated for worms against the mean number in those that were untreated. A sample of 14 worm-infected lambs was randomly divided into 2 groups. Seven were injected with the drug and the remainder were left untreated. After a six-month period, the lambs were slaughtered and the following worm counts were recorded:

Drug-treated	18	43	28	50	16	32	13
Untreated	40	54	26	63	21	37	39

Test the null hypothesis that the treatment is ineffective against the alternative that it decrease the mean number of worms in lambs. Assume normal distribution with equal variance and use $\alpha = 0.05$.