

2. Let $\vec{b} \in \mathbb{R}^m$, $A \in M_{m \times n}(\mathbb{R})$ and let R be any row echelon form of A . Which of the following statements must be true?
 - (a) If the linear system of equations $A\vec{x} = \vec{b}$ has infinitely many solutions, then R has at least one row of zeros.
 - (b) If R has at least one row of zeros and the linear system of equations $A\vec{x} = \vec{b}$ is consistent, then the system has infinitely many solutions.
 - (c) If the linear system of equations $A\vec{x} = \vec{b}$ is inconsistent, then R has at least one row of zeros.
 - (d) The linear systems of equations $A\vec{x} = \vec{b}$ and $R\vec{x} = \vec{b}$ have the same set of solutions.
 - (e) None of the above.

3. Which of the following statements are true for any subspace \mathbb{S} of \mathbb{R}^n ?
 - (a) If $\dim(\mathbb{S}) = k$, then every spanning set for \mathbb{S} contains exactly k vectors.
 - (b) If $\dim(\mathbb{S}) = k$, then every linearly dependent set of vectors in \mathbb{S} contains more than k vectors.
 - (c) \mathbb{S} is closed under linear combinations.
 - (d) If $\mathbb{S} \neq \{\vec{0}\}$, then there are infinitely many different bases for \mathbb{S} .
 - (e) None of the above.

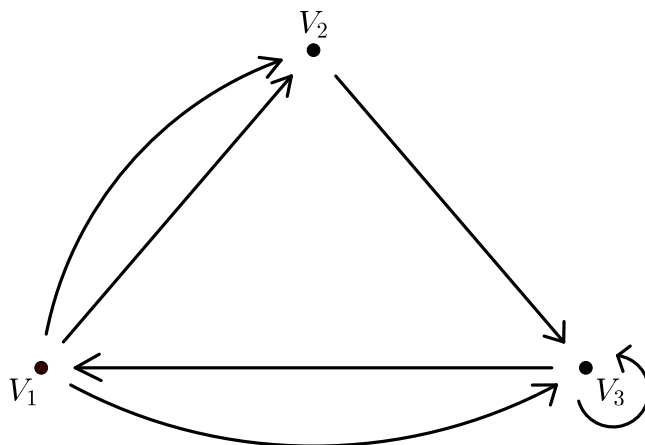
4. Which of the following statements are true for any $A \in M_{m \times n}(\mathbb{R})$?
 - (a) The rank of A is the number of nonzero rows in any row echelon form of A .
 - (b) If $B \in M_{m \times n}(\mathbb{R})$ can be obtained by performing an elementary row operation on A , then $\text{rank}(A) = \text{rank}(B)$.
 - (c) If $A\vec{x} = \vec{b}$ is consistent for some $\vec{b} \in \mathbb{R}^m$, then $\text{rank}(A) = m$.
 - (d) For any $\vec{b} \in \mathbb{R}^m$, $\text{rank}([A \mid \vec{b}]) \geq \text{rank}(A)$.
 - (e) None of the above.

5. Which of the following statements are true for any square matrix $A \in M_{n \times n}(\mathbb{R})$?
 - (a) If $\text{Col}(A) = \text{Row}(A)$, then A is symmetric.
 - (b) If A is symmetric, then $\text{Col}(A) = \text{Row}(A)$.
 - (c) If $\text{Null}(A) = \{\vec{0}\}$, then $\text{Col}(A) = \mathbb{R}^n$.
 - (d) $\vec{0} \in \text{Col}(A) \cap \text{Row}(A)$.
 - (e) None of the above.

6. Which of the following statements hold for any two invertible matrices $A, B \in M_{n \times n}(\mathbb{R})$?

- (a) $(A + B)^{-1} = A^{-1} + B^{-1}$.
- (b) $(AB)^{-1} = A^{-1}B^{-1}$.
- (c) $A^{-1} + B^{-1} = B^{-1} + A^{-1}$.
- (d) $A^{-1}B^{-1} = B^{-1}A^{-1}$.
- (e) None of the above.

7. Consider the following directed graph on the three vertices V_1, V_2 and V_3 .



How many distinct 4-edged paths are there from V_1 to V_2 .

- (a) 2.
- (b) 6.
- (c) 16.
- (d) 4.
- (e) None of the above.

8. Consider the subspace $\mathbb{S} = \text{Span} \left\{ \begin{bmatrix} 1 \\ -3 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix} \right\}$ of \mathbb{R}^4 . Which of the following

sets are bases for \mathbb{S} ?

(a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\}.$

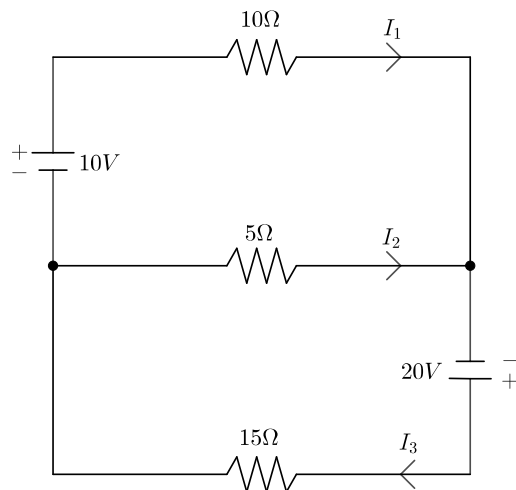
(b) $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ 2 \\ -1 \end{bmatrix} \right\}.$

(c) $\left\{ \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 1 \\ 2 \end{bmatrix} \right\}.$

(d) $\left\{ \begin{bmatrix} -1 \\ 3 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -2 \\ 0 \\ 5 \end{bmatrix} \right\}.$

(e) None of the above.

9. Consider the electrical network shown below.



Solving for the currents I_1 , I_2 and I_3 gives

- (a) $I_1 = \frac{4}{5}$ amps, $I_2 = \frac{2}{5}$ amps, $I_3 = \frac{6}{5}$ amps.
- (b) $I_1 = 0$ amps, $I_2 = -2$ amps, $I_3 = 2$ amps.
- (c) $I_1 = \frac{12}{11}$ amps, $I_2 = \frac{2}{11}$ amps, $I_3 = \frac{14}{11}$ amps.
- (d) $I_1 = \frac{12}{5}$ amps, $I_2 = \frac{14}{5}$ amps, $I_3 = \frac{2}{5}$ amps.
- (e) None of the above.

Part 2: Written Response (19 marks total)

Instructions: The following five questions should each be answered on a separate sheet of paper, and uploaded to Crowdmark following the usual Written Assignment Instructions. Failure to follow those instructions can result in a grade of zero on any affected questions.

1. (5 marks) Let $a, b \in \mathbb{R}$ be constants. Let $\vec{x}, \vec{y} \in \mathbb{R}^4$ where

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3a - 4 \\ 10b - 4 \end{bmatrix} \quad \text{and} \quad \vec{y} = \begin{bmatrix} a \\ 1 + 2a \\ -2 - 4a + 3a^2 \\ 10ab + 5b - 4a \end{bmatrix}.$$

Find two nonzero and non-parallel vectors \vec{u} and \vec{v} such that \vec{u} is orthogonal to both \vec{x} and \vec{y} and that \vec{v} is orthogonal to both \vec{x} and \vec{y} . Clearly show how you derive your solution.

2. (4 marks) Let \mathbb{S}_1 and \mathbb{S}_2 be subspaces of \mathbb{R}^2 . Consider the set

$$\mathbb{T} = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbb{R}^4 \mid \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \in \mathbb{S}_1 \quad \text{and} \quad \begin{bmatrix} x_3 \\ x_4 \end{bmatrix} \in \mathbb{S}_2 \right\}.$$

Prove that \mathbb{T} is a subspace of \mathbb{R}^4 .

3. (5 marks)

(a) Solve for the matrix A if

$$\begin{bmatrix} 3 & 3 & -3 \\ 4 & 8 & -4 \\ -3 & -5 & 4 \end{bmatrix}^T = \left(2A + \begin{bmatrix} 1 & 3 & -5 \\ 2 & -6 & 2 \\ 4 & 0 & -8 \end{bmatrix} \right)^{-1}.$$

(b) Let $\vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5 \in \mathbb{R}^3$ where

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad \text{and} \quad \vec{v}_5 = \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix}$$

Find \vec{v}_3 and \vec{v}_4 if $B = \begin{bmatrix} \vec{v}_1 & \vec{v}_2 & \vec{v}_3 & \vec{v}_4 & \vec{v}_5 \end{bmatrix}$ and the RREF of B is

$$R = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

You may not perform any Elementary Row Operations on B or R .

4. (**4 marks**) Let $\{\vec{v}_1, \dots, \vec{v}_n\}$ be a basis for \mathbb{R}^n and let $A_1, A_2 \in M_{m \times n}(\mathbb{R})$ be such that $A_1 \vec{v}_i = A_2 \vec{v}_i$ for $i = 1, \dots, n$. Prove that $A_1 = A_2$.
5. (**1 mark**) Write $\frac{3+j}{2-j}$ in standard form.