

Submission via Turnitin

Be as concise as possible. Total: 25 points

Part A. Multiple Choice Questions (5 points, 1 point each question)

- A1. When we conduct t -test, we have to be cautious about the structure of our data. Which in the following is not a concern?
- A) single sample vs. paired groups
 - B) correlated groups
 - C) independent groups
 - D) mutually exclusive groups
- A2. For a simple linear regression with two variables X and Y , we can estimate the slope in the linear regression line by Pearson correlation r between X and Y , and the standard deviation of S_X and S_Y . If we know $r = 0.75$, $S_X = 5$, and $S_Y = 8$, what is the estimated slope (b) of the line $Y = a + bX$?
- A) 3.75
 - B) 0.469
 - C) 1.2
 - D) 1.6
- A3. In linear regression, if we know $Y = 25 + 1.3 X$. Given a *change* in the independent variable X of 20 units, what is the corresponding *change* in the dependent variable Y ?
- A) 51 units
 - B) 26 units
 - C) 25 units
 - D) 1.3 units
- A4. Power of test could be affected by a few factors, what of the following is NOT one of them?
- A) sample size N
 - B) sample variability S
 - C) confidence interval
 - D) the size of the real effect of the independent variable
- A5. If two samples are selected from the same population, under what circumstances will the two samples have exactly the same t statistic?
- A) If the sample size (n) is the same for both samples.
 - B) If the samples are the same size and have the same mean.

- C) If the samples are the same size and have the same mean and have the same sample variance.
- D) None of the other options are correct.

Part B. Short Essay Questions. Please use hand calculation to show the steps. (20 points total)

B1. A nutrition store in the mall is selling Memory Booster which is a concoction of chicken essence, herbs and minerals that is intended to improve memory performance. To test the effectiveness of the Memory Booster, a researcher obtained a sample of $n = 30$ participants and divided them into two groups with equal sample size. In one group, the researcher asked each participant to take the suggested dosage each day for 4 weeks. In the other group, the participants did NOT take any of the Memory Boost. At the end of the four-week period, each individual took a standardized memory test. The scores from the Memory Boost sample produced a mean of $M_1 = 26$ with $SS = 960$. In the group that did not take the Memory Boost, the mean is $M_2 = 20$ and $SS = 1100$.

- a. Do the sample data support the conclusion that the Memory Booster has a significant effect? Test with $\alpha = .05$. (5 points)

- b. Compute effect size of the Memory Boost. (3 points)

Locate the critical region:

$$df_1 = 15 - 1 = 14, df_2 = 15 - 1 = 14$$

$$s_p^2 = \frac{SS_1 + SS_2}{df_1 + df_2} = \frac{s_1^2 \times df_1 + s_2^2 \times df_2}{df_1 + df_2}$$

Since $SS_1 = 960$ and $SS_2 = 1100$,

$$s_p^2 = (960 + 1100) / (14 + 14) = 73.5714$$

$$\text{Cohen's } d = \frac{(\bar{X}_1 - \bar{X}_2)}{\sqrt{s_p^2}}$$

$$\text{Cohen's } d = \frac{26 - 20}{\sqrt{73.5714}} = 0.6995 = 0.70 \text{ (2. d. p)}$$

The effect size is 0.70 standard deviation units. Memory boost has a medium effect on improving memory performance

c. What is your conclusion of the study? (2 points)

B2. Amy was interested in studying if third grade students' math performance was related to their IQ. She then chose one of the schools in the community and conducted her study on the third grade students in a class. She first tested the students' IQ and then looked into their most recent math test scores on a closed book exam. Here is the list of the scores she got.

Student No.	IQ score	Math score
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1	105	80
2	85	72
3	95	75
4	140	93
5	70	61
6	100	66
7	90	71
8	110	80
9	120	90
10	95	65

Please answer the following questions:

- a. Amy is interested in finding out if there is any correlation in the IQ and math score. Can you help her? What is the correlation? Is it significant? (5 marks)

Yes, I can help her by using the Pearson's correlation coefficient to solve this question. Pearson's correlation coefficient will help me find out if there is any linear relationship (correlation) between the IQ score and math score.

Step 1: Label the X and Y variables:

Let the X-variable be IQ score

Let the Y-variable be Math score

Step 2: Find the mean values of X and Y

To find the mean of IQ score (M_X) :

$$(105 + 85 + 95 + 140 + 70 + 100 + 90 + 110 + 120 + 95) / 10 = 101$$

To find the mean of Math Score (M_Y) :

$$(80 + 72 + 75 + 93 + 61 + 66 + 71 + 80 + 90 + 65) / 10 = 75.3$$

Step 3: Measure the variability of X and Y separately:

$$SS_X = \sum (X - M_X)^2$$

$$= (105 - 101)^2 + (85 - 101)^2 + (95 - 101)^2 + (140 - 101)^2 + (70 - 101)^2 + (100 - 101)^2 + (90 - 101)^2 + (110 - 101)^2 + (120 - 101)^2 + (95 - 101)^2$$

$$= 3390$$

$$SS_Y = \sum (Y - M_Y)^2$$

$$= (80 - 75.3)^2 + (72 - 75.3)^2 + (75 - 75.3)^2 + (93 - 75.3)^2 + (61 - 75.3)^2 + (66 - 75.3)^2 + (71 - 75.3)^2 + (80 - 75.3)^2 + (90 - 75.3)^2 + (65 - 75.3)^2$$

$$= 1000.1$$

Step 4: Measure the covariability of X and Y together

$$SP = \sum (X - M_X)(Y - M_Y)$$

$$= (105 - 101)(80 - 75.3) + (85 - 101)(72 - 75.3) + (95 - 101)(75 - 75.3) + \\ (140 - 101)(93 - 75.3) + (70 - 101)(61 - 75.3) + (100 - 101)(66 - 75.3) + \\ (90 - 101)(71 - 75.3) + (110 - 101)(80 - 75.3) + (120 - 101)(90 - 75.3) + \\ (95 - 101)(65 - 75.3)$$

$$= 1647$$

Step 5 : Solve Pearson Correlation (r):

$$r = \frac{SP}{\sqrt{SS_X SS_Y}} = \frac{1647}{\sqrt{(3390)(1000.1)}} = 0.8944 = 0.89 \text{ (2.d.p)}$$

For the subsequent steps, I will be testing the significance for the correlation by using a non-directional test

Step 6: State the hypotheses

The null hypothesis H_0 : Population correlation $\rho = 0$

The Alternative Hypothesis H_1 : Population correlation $\rho \neq 0$

Step 7: From the t statistics,

$$t_{obt} = \frac{r_{obt} - \rho}{s_r} = \frac{r_{obt} - \rho}{\sqrt{\frac{1 - r_{obt}^2}{N - 2}}} = \frac{0.8944 - 0}{\sqrt{\frac{1 - (0.8944)^2}{10 - 2}}} = \frac{0.8944}{0.1581} = 5.6571 = 5.657 \text{ (3.d.p)}$$

From the t table, $df = 10 - 2 = 8$ and $\alpha = 0.05$ (2-tailed), I found that $t_{crit} = 2.306$. Since $|t_{obt}| > 2.306$, I decide to reject H_0 and conclude that there is a significant positive correlation between IQ score and Math Score in the population.

- b. She then wants to predict math performance from IQ. Can you find the prediction equation for her? (3 points)

Yes, I can find the prediction equation by using Linear Regression .

Equation for the regression line:

$$\hat{Y} = bX + a \quad \text{where } b = \frac{SP}{SS_X} \quad \text{and} \quad a = M_Y - bM_X$$

Since $SP = 1647$ and $SS_X = 3390$,

I can deduce that :

$$b = \frac{SP}{SS_X} = \frac{1647}{3390} = 0.4858 = 0.486 \text{ (3.d.p)}$$

$$a = M_Y - bM_X = 75.3 - 0.486 \times 101 = 26.214 = 26.21 \text{ (2.d.p)}$$

Therefore, the equation is:

$$\hat{Y} = \mathbf{0.486 X + 26.21}$$

- c. One new student joined the class, but his math score was not on record. However, Amy found that his IQ score was 20 points higher than the student average in class. What would be the student's math score? (2 points)

$$\text{The new IQ score of the new student} = 101 + 20 = 121$$

New student's Math score :

$$\hat{Y} = 0.486 X + 26.21 = 0.486 (121) + 26.21 = 85.016 = 85.02 \text{ (2.d.p)}$$