**Fall 2020 Take home midterm**

**Instructions**: Free feel to use R to do computations. The test has a total of 100 points. You can write your answers in papers and submit in pdf format. You are encouraged to type your solutions in R markdown pdf file.

Some critical values you may use: *z*0.975 =1.96 , χ0.95,92 =16.9 , χ0.95,102 =18.3 , χ0.95,112 =19.7 ,

Pr(χ92 > 8.45)= 0.49, Pr(χ102 > 4.58)= 0.92, Pr(χ112 > 9.85)= 0.54.

1. (**20 points**) True/False questions. Circle your answer.
   1. True or False : Exact tests, e.g., derived using level α (likelihood ratio tests) LRTs, guarantee the Type I error rates are always less than the desired αlevel; Sometimes they may be conservative.
   2. True or False: Power function gives us the probability of rejecting null hypotheses for every possible value of the parameter.
   3. True or False: Power is the probability of correctly rejecting null hypothesis when the alternative hypothesis is true.
   4. True or False: Type II error probability equals to one minus Type I error probability.

1. (**40 points**) We collected a sample of 6 high school students and recorded their scores on standardized tests in the following table including tests of reading (read) and writing (write).

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Student | 1 | 2 | 3 | 4 | 5 | 6 |
| read | 34 | 39 | 44 | 50 | 55 | 60 |
| write | 33 | 44 | 50 | 46 | 61 | 62 |

* 1. (**5**) Based on the data set, draw a scatterplot (X-Y) of test scores and briefly describe the pattern you see in the plot. In the plot, set read on the x-axis. 2.2 (**10**) Suppose that the random variable *Y*1,...,*Yn* satisfy

*Yi* =β ε*xi* + *i* , *i* =1,..., ,*n*

where *x*1,...,*xn* are fixed constants, and ε ε1,..., *n* are independent and identically distributed

(i.i.d.) normal random variables *N*(0,σ2 ) , where σ2 is unknown. That says *Yi*  *N* (β σ*xi* , 2), *i* =1,..., .*n* Find the MLEs of β and σ2 given the pair observations

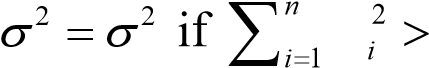
(*Y x*1, 1),...,(*Y xn*, *n* ).

2.3 (**10**) Based on (2.2), show that the MLE of β is an unbiased estimator of β.

2.4 (**5**) Note that it is natural that student gets 0 score on writing test due to lack of reading ability. Combing the association between read and write you observed from 2.1, it is reasonable to build a linear regression model using the paired data (write, read) without intercept. In this model, you treat write as a response variable (Y) and read as an independent variable (X). Based on the data set and the result of 2.3, write down the estimated mean regression equation, e.g., *Y* =βˆ*x* .

2.5 (**5**) Find the variance of the MLE βˆ and the distribution of the MLE βˆ .

2.6 (**5**) Construct the asymptotic 95% confidence interval for β using the result of (2.5), and perform the asymptotic level α test for *H*0 :β= 0 vs. *H*1 :β≠ 0 using the confidence interval and briefly state your conclusion.

1. (**40 points**) A manufacturer of hard safety hats for construction workers is concerned about the variation of the forces its helmets transmit to wearers when subjected to an external force. For simplicity, we assume that the measurements of the forces of *n* helmets in an experiment that transmit to wearers, *X*1,..., *Xn* , are a random sample from *N*(0,σ2 ) , where σ2 is unknown. Consider testing this simple hypotheses *H*0 :σ σ2 = 02 vs. *H*1 :σ σ2 = 12 , where the known constants satisfy σ σ02 < 12 and parameter space is Ω={σσ02, 12} (You already know that this statement of hypotheses is equivalent to that *H*0 :σ σ2 = 02 vs. *H*1 :σ σ2 > 02 ).
   1. (**10**) Using LRT to show that we reject *H*0 : 0 *X C* for some constant *C*.
   2. (**5**) Is the above obtained LRT a uniformly most powerful test? If yes, briefly state your reason.
   3. (**10**) Given the significance level α, find appropriate value of *C* to make the LRT obtain in

3.1 a level α test.

* 1. (**10**) Suppose *H*0 :σ2 =1 vs. *H*1 :σ2 = 4 , sample size *n*=10 and α= 0.05. Find the value of *C* and the power of test when *H*1 :σ2 = 4 is true.
  2. (**5**) Calculate the Type II error probability when *H*1 :σ2 = 4 is true.