

- 2) Download the data `production2.dta` from Collab. This data set contains data on output (value added) and inputs at the industry level for 459 industries in 1958 and 1993. Suppose the relationship between output and inputs is described by a Cobb-Douglas production function

$$Y_i = AK_i^\alpha L_i^\beta$$

where Y_i is a measure of output, K_i is the capital stock, and L_i is employment. Answer all questions for the year 1958 only.

- a) Transform the production function to a linear equation by taking logs. Estimate the parameters α and β by an OLS regression using total value added as your measure of output.
- b) Test whether your estimates are consistent with the production function exhibiting constant returns to scale, i.e.,

$$H_0 : \alpha + \beta = 1$$

against the alternative

$$H_1 : \alpha + \beta \neq 1$$

Do you reject the hypothesis at the 5% significance level? What is the p -value of your test?

- c) An alternative way to test the hypothesis of constant returns to scale to impose this restriction on the parameters and transform your regression model. Derive the necessary transformation, and show how the constant returns hypothesis amounts to a t -test in this transformed model. Carry out this test. Verify that your result matches what you found in (b).
- d) An alternative way of specifying the production function is to define Y_i as the total value of shipments and to add materials as a factor of production. Test whether this three factor production function exhibits constant returns to scale.
- e) Finally, also add energy as a fourth factor of production to the model in (d).
1. Is energy a significant factor of production?
 2. Does this model exhibit constant returns to scale?
- f) Explain for which specification of the production function the test for constant returns to scale makes the most sense (i.e. think about the economic content of the idea of constant returns).
- g) Discuss the interpretation of the estimated production function parameters you obtained in this problem set. In particular, are you worried that the estimates may not actually reflect the production function parameters (are you necessarily estimating the causal

effect of a firms changing their input levels and outputs responding to that)? Think about the role of A in the production function in this context.