Assignment#2

Instructions:

1. Do not copy. We will not grade plagiarized assignments.

2. Publish the MATLAB script as PDF with all functions, outputs and plots included. You can also submit the livescripts.

3. Save the PDF file or livescript with "RollNo_ProbNo.pdf" or "RollNo_ProbNo.mlx". DO NOT ZIP the files/folders. Upload EACH pdf/script in the Google Classroom.

4. Use of MATLAB intrinsic functions is not recommended unless otherwise mentioned in the problem. Anyhow you may use intrinsic functions to check your results.

5. *If you have any query, please post your specific queries on the Google Classroom.*

Problem 1.

A. Determine the multiple real roots of the given equation by

a) Graphically

b) Using Newton-Raphson method to within the tolerance of 0.01%

$$f(x) = 0.5x^3 - 4x^2 + 5.5x - 1$$

B. Determine the roots of following simultaneous nonlinear equations using multiple Newton-Raphson method. Employ initial guesses of x = y = 1.2

$$y = -x^2 + x + 0.75$$

$$y + 5xy = x$$

C. Determine the roots of following simultaneous nonlinear equations using multiple Newton-Raphson method. Use a graphical approach to obtain initial guesses.

$$(x-4)^2 + (y-4)^2 = 5$$

$$x^2 + y^2 = 16$$

Problem 2.

Write a $MATLAB$ function $t = tr(A)$ which computes the trace of a given matrix A. The trace of a matrix is given by the sum of its diagonal elements. Test to make sure that A is a square matrix.

Problem 3.

Given matrix $A = 6 * eye(5) - ones(5,5)$. Use Gauss Elimination to find its inverse, A^{-1}. Verify your answer using left division operator in $MATLAB$. And verify $A * A^{-1} = I$.

Problem 4.

For the following system of linear algebraic equations:

$$\begin{aligned} 9x_1 + 4x_2 - 3x_3 + 19x_4 + 11x_5 &= -12 \\ -8x_1 + x_2 + 2x_3 + 8x_4 - 12x_5 &= -8 \\ 5x_1 + x_2 - 7x_3 + 3x_4 + 4x_5 &= -2 \\ 3x_1 - 2x_2 + x_3 - 5x_4 + 6x_5 &= 5 \\ 7x_1 - 6x_2 + 3x_3 - 2x_4 + 3x_5 &= 4 \end{aligned}$$

a. The coefficient matrix, A can be written as $A = L * U$ where L is lower-triangular matrix and has only *ones* on its diagonal and U is upper-triangular matrix. Use Gauss Elimination (without pivoting) to find L and U.

b. Verify whether $L * U = A$, the original coefficient matrix.

c. Find determinant of A using Gauss Elimination (without pivoting). Show that $det(L * U) = det(L) * det(U) = det(A)$.

d. Find the solution of the given set of linear algebraic equations using Gauss Elimination (without pivoting).

Problem 5.

Repeat 4a, 4b, 4c and, 4d using Gauss Elimination with partial pivoting. How many row interchanges has been made?

Problem 6.

Repeat 4a, 4b, 4c and, 4d using Gauss Elimination with scaled partial pivoting. How many row interchanges has been made? Compare the accuracy of P4, P5 and, P6.

Problem 7.

Hilbert matrix: The $n \times n$ Hilbert matrices are defined by

$$H(i,j) = 1/(i+j-1),\ 1 \leq i,\ j \leq n.$$

a. Use this in a program that displays the Hilbert matrix of order-20 and check with the $MATLAB$ command $\gg H = hilb(20)$.

b. What is the condition number of the 20×20 Hilbert matrix, H_{20}.

c. Solve system of equations $Hx = [1\ 1\ \cdots\ 1]^T$ where H is a 20×20 Hilbert matrix using LU decomposition.

d. Now solve $Hx = [0.99\ 0.99\ \cdots\ 0.99]$. Discuss the accuracy of the results.

Problem 8.

Using $MATLAB$ command $diag$, build a tridiagonal matrix T as follows:

$\gg a = ones(4, 1);$

$\gg b = 5 * ones(5, 1);$

$\gg c =- ones(4, 1);$

$\gg T = diag(a,-1) + diag(b) + diag(c, 1);$

Write a LU decomposition function to factor the matrix T. Verify that $T = L * U$.