

**Graduate School of Business
UNIVERSITY OF CAPE TOWN**

Quantitative Methods in Development Finance
MCom in Development Finance

Course Code: GSB5104F

August, 2016

Time: 3 hours

Instructions: Attempt all questions

Note: This is a closed book exams. You are ONLY ALLOWED TWO A4 size papers of **HAND WRITTEN** notes into the exams room. No text books or other unauthorised material are allowed into the room.

Question 1**(4 marks)**

Suppose X = the number of cars owned by a family in South Africa. The probability distribution of X is shown in the table below.

X	0	1	2	3
Probability	0.56	0.23	0.12	0.09

- (a) What is the chance that a family owns more than one car?
- (b) Suppose you choose two families at random. What is the chance that they each own one car? (That is family A owns a car and family B owns a car)

Question 2**(8 marks)**

A construction company has submitted bids on two separate Provincial contracts, A and B . The company feels that it has a 60% chance of winning contract A , and a 50% chance of winning contract B . Furthermore, the company believes that it has an 80% chance of winning contract A if it wins contract B .

- (a) What is the probability that the company will win both contracts?
- (b) What is the probability that the company will win at least one of the two contracts?
- (c) If the company wins contract B , what is the probability that it will not win contract A ?
- (d) What is the probability that the company will win neither contract?

Question 3**(7 marks)**

The monthly sales at a bookstore have a mean of R50 000 and a standard deviation of R6000. Profits are calculated by multiplying sales by 40% and subtracting fixed costs of R12 000. Find the mean and standard deviation of monthly profits. What assumptions have you made before proceeding to tackle the question?

Question 4**(7 marks)**

A bank has determined that the monthly balances of the saving accounts of its customers are normally distributed with an average balance of R1200 and a standard deviation of R250.

- (a) What proportion of customers have monthly balances less than R1000?
- (b) What proportion of customers have monthly balances more than R1125?
- (c) What proportion of customers have monthly balances between R950 and R1075?

Question 5**(6 marks)**

A production filling operation has a historical standard deviation of 6 ounces. When in proper adjustment, the mean filling weight for the production process is 50 ounces. A quality control inspector periodically selects, at random, 36 containers and uses the sample mean filling weight to see if the process is in proper adjustment.

- State the null and alternative hypotheses;
- Using a standardized test statistic, test the hypothesis at the 5% level of significance if the sample mean filling weight is 48.6 ounces;
- Develop a 95% confidence interval and use it to test the hypothesis.

Question 6**(8 marks)**

At a recent Elton John concert, a survey was conducted that asked a random sample of 20 people their age and how many concerts they have attended since the first of the year. It is suspected that older concert goers tend to go to more of his concerts in one year than younger concert goers. The data and analysis are shown below.

Age	62	57	40	49	67	54	43	65	54	41
Number of Concerts	6	5	4	3	5	5	2	6	3	1

Age	44	48	55	60	59	63	69	40	38	52
Number of Concerts	3	2	4	5	4	5	4	2	1	3

An Excel output follows:

SUMMARY OUTPUT

<i>Regression Statistics</i>	
Multiple R	0.80203
R Square	0.64326
Adjusted R Square	0.62344
Standard Error	0.93965
Observations	20

DESCRIPTIVE STATISTICS

<i>Age</i>		<i>Concerts</i>	
Mean	53	Mean	3.65
Standard Error	2.1849	Standard Error	0.3424
Standard Deviation	9.7711	Standard Deviation	1.5313
Sample Variance	95.4737	Sample Variance	2.3447
Count	20	Count	20

SPEARMAN RANK CORRELATION COEFFICIENT=0.8306

ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Significance F</i>
Regression	1	28.65711	28.65711	32.45653	2.1082E-05
Residual	18	15.89289	0.88294		
Total	19	44.55			

	<i>Coefficients</i>	<i>Standard Error</i>	<i>t Stat</i>	<i>P-value</i>	<i>Lower 95%</i>	<i>Upper 95%</i>
Intercept	-3.01152	1.18802	-2.53491	0.02074	-5.50746	-0.5156
Age	0.12569	0.02206	5.69706	0.00002	0.07934	0.1720

- (a) Conduct a test of the population coefficient of correlation to determine, at the 5% significance level, whether a positive linear relationship exists between age and number of concerts attended;
- (b) Conduct a test of the population slope to determine, at the 5% significance level, whether a positive linear relationship exists between age and number of concerts attended.

Question 7

(10 marks)

A real estate builder wishes to determine how house size is influenced by family income, family size, and education of the head of household. House size is measured in hundreds of square feet, income is measured in thousands of Rands, and education is measured in years. A partial computer output is shown below.

SUMMARY OUTPUT

Regression Statistics

Multiple R	0.865
R Square	0.748
Adjusted R Square	0.726
Standard Error	5.195
Observations	50

ANOVA

	<i>df</i>	<i>SS</i>	<i>MS</i>	<i>F</i>	<i>Signif F</i>
Regression		3605.7736	901.4434		0.0001
Residual		1214.2264	26.9828		
Total	49	4820.0000			

	<i>Coeff.</i>	<i>St. Error</i>	<i>t Stat</i>	<i>P-value</i>
Intercept	-1.6335	5.8078	-0.281	0.7798
Family Income	0.4485	0.1137	3.9545	0.0003
Family Size	4.2615	0.8062	5.286	0.0001
Education	-0.6517	0.4319	-1.509	0.1383

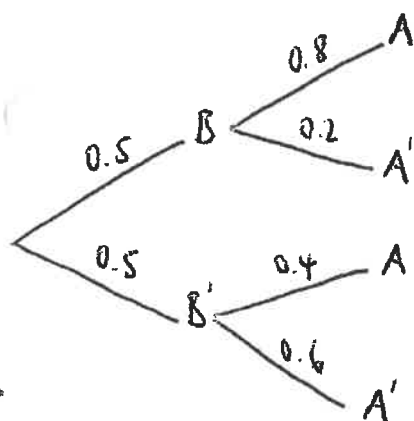
- (a) What percentage of the variability, in house size, is explained by this model?
- (b) Which of the independent variables, in the model, are significant at the 2% level?
- (c) When the builder used a simple linear regression model with house size as the dependent variable and education as the independent variable, he obtained an R-square value of 23.0%. What additional percentage of the total variation in house size has been explained by including family size and income in the multiple regression?
- (d) What is the predicted house size for an individual earning a monthly income of R40 000, having a family size of 4, and having 13 years of education?

Question 1

$$a) P(X > 1) = P(X=2) + P(X=3) = 0.12 + 0.09 = 0.21$$

$$b) P(X_A=1 \text{ and } X_B=1) = P(X_A=1) \times P(X_B=1) = 0.23 \times 0.23 = 0.0529$$

Question 2



$$0.5 \times 0.8 + 0.5 \times A/B' = 0.6$$

$$\therefore A/B' = \frac{0.6 - 0.5 \times 0.8}{0.5} = 0.4$$

$$a) P(A \text{ and } B) = 0.5 \times 0.8 = 0.4$$

$$b) 1 - P(B' \text{ and } A') = 1 - 0.5 \times 0.6 = 0.7$$

(or $0.5 \times 0.8 + 0.5 \times 0.2 + 0.5 \times 0.4 = 0.7$)

$$c) P(A'/B) = \frac{P(A' \text{ and } B)}{P(B)} = \frac{0.5 \times 0.2}{0.5} = 0.2$$

$$d) P(A' \text{ and } B') = 0.5 \times 0.6 = 0.3$$

Question 3

Assumption: Sales(S) is a random variable

$$P = 0.4S - 12000$$

$$E(P) = E(0.4S - 12000)$$

$$= E(0.4S) - E(12000)$$

$$= 0.4E(S) - 12000$$

$$= 0.4 \times 50000 - 12000 = 8000$$

$$\text{Var}(P) = \text{Var}(0.4S - 12000)$$

$$= \text{Var}(0.4S) = 0.4^2 \text{Var}(S)$$

$$= 0.4^2 \times 6000^2$$

$$\therefore \sigma_P = 0.4 \times 6000 = 2400$$

Question 4

$$\mu = 1200, \sigma = 250$$

$$a) P(X < 1000)$$

$$= P\left(Z < \frac{1000 - 1200}{250}\right) = P(Z < -0.8)$$

$$= 0.2119$$

$$b) P(X > 1125) = P\left(Z > \frac{1125 - 1200}{250}\right)$$

$$= P(Z > -0.3) = 1 - P(Z < -0.3) = 1 - 0.3821$$

$$= 0.6179$$

$$c) P(950 < X < 1075) = P(X < 1075) - P(X < 950)$$

$$= P\left(Z < \frac{1075 - 1200}{250}\right) - P\left(Z < \frac{950 - 1200}{250}\right)$$

$$= P(Z < -0.5) - P(Z < -1) = 0.3085 - 0.1587 = 0.1498$$

Question 5

a) $H_0: \mu = 50$
 $H_1: \mu \neq 50$

b) ~~$t = \frac{\bar{X} - \mu}{s/\sqrt{n}} = \frac{48.6 - 50}{6/\sqrt{36}}$~~

b) $z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} = \frac{48.6 - 50}{6/\sqrt{36}} = -1.4$

Reject H_0 if $z < -z_{0.05}$ or $z > z_{0.05}$

$z_{0.05} = 1.96$

Since $z > -z_{0.05}$, we ^{do not} reject the null hypothesis.
Therefore, there is insufficient evidence to infer that $\mu \neq 50$.

c) $(\bar{X} - z_{0.05} \times \frac{\sigma}{\sqrt{n}} ; \bar{X} + z_{0.05} \frac{\sigma}{\sqrt{n}})$

$= (48.6 - 1.96 \times \frac{6}{\sqrt{36}} ; 48.6 + 1.96 \times \frac{6}{\sqrt{36}})$

$= (48.6 - 1.96 ; 48.6 + 1.96)$

$= (46.64 ; 50.56)$

Question 6

a) $H_0: \rho = 0$
 $H_1: \rho > 0$, $t = r \sqrt{\frac{n-2}{1-r^2}} = \sqrt{0.64326} \sqrt{\frac{18}{1-0.64326}}$
 $= 5.70$

Reject H_0 if $t > t_{0.05} = 1.734$ where $v = 18$

Since $t > t_{0.05}$, reject H_0 . Therefore there is sufficient evidence to infer a positive linear relationship exists between age and number of concerts attended.

b) $H_0: \beta = 0$
 $H_1: \beta \neq 0$

Reject H_0 if p-value is less than 0.05
p-value = 0.00002

Since reject p-value < 0.05, we reject.

Therefore there is sufficient evidence to infer that a positive linear relationship exists between age and number of concerts attended.

Question 7

a) $R^2 = 0.748$

b) Family income and Family size

Since p-value corresponding to family size and family income are both less than 0.02.

$H_0: \beta_i = 0$ for $i = 1, 2, 3$
 $H_1: \beta_i \neq 0$

c) $74.8\% - 23\% = 51.8\%$

d) Updated regression model:

$\hat{y} = -1.6335 + 0.4485x_1 + 4.2615x_2$

$\hat{y} = -1.6335 + 0.4485 \times 40 + 4.2615 \times 4$
 $= 33.3525 \therefore 3,335.25 \text{ ft}^2$

Appendix: Exam 2016 Q7(b)

$H_0: \beta_1 = 0$ reject H_0 if $p\text{-value} < 0.02$
 $H_1: \beta_1 \neq 0$

Since $p\text{-value} = 0.0003 < 0.02$, reject the H_0 .

Therefore there is sufficient evidence to infer that house size and family income are related.

$H_0: \beta_2 = 0$ reject H_0 if $p\text{-value} < 0.02$
 $H_1: \beta_2 \neq 0$

Since $p\text{-value} = 0.0001 < 0.02$, we reject H_0 .

Therefore there is sufficient evidence to infer that house size and family size are linearly related.

$H_0: \beta_3 = 0$ reject H_0 if $p\text{-value} < 0.02$
 $H_1: \beta_3 \neq 0$

Since $p\text{-value} = 0.1383 > 0.02$, we do not reject the H_0 .

Therefore there is insufficient evidence to infer that house size and education are linearly related.

