

the preceding players. Let 1-8 represent the players' heights. An example of a valid ordering: 4, 5, 6, 3, 2, 7, 1, 8. An example of an invalid ordering: 1, 2, 3, 4, 5, 6, 8, 7. (invalid since 7 is neither taller or shorter than all the preceding players). How many such orderings exist?

- (d) To keep viewership up after the tournament finishes, the council plans an All-Star match. The 16 greatest players in the league were chosen, including Oski and a tree..? Oski refuses to play on the same team as the tree. How many ways can the 16 players be distributed into two teams of 8 players such that Oski and the tree are in opposite teams?
- (e) Provide an explanation for the following combinatorial identity. Hint: Solve the previous part using another method. Those two methods should correspond to the two sides of the equality.

$$\binom{n}{r} - \binom{n-2}{r-2} - \binom{n-2}{r} = 2\binom{n-2}{r-1}$$

5 School Carpool

- (a) n males and n females apply to EECS within UC Berkeley. The EECS department only has n seats available. In how many ways can it admit students? Use the above story for a combinatorial argument to prove the following identity:

$$\binom{2n}{n} = \binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2$$

- (b) Among the n admitted students, there is at least one male and at least one female. On the first day, the admitted students decide to carpool to school. The male(s) get in one car, and the female(s) get in another car. Use the above story for a combinatorial argument to prove the following identity:

$$\sum_{k=1}^{n-1} k \cdot (n-k) \cdot \binom{n}{k}^2 = n^2 \cdot \binom{2n-2}{n-2}$$

(Hint: Consider the ways that students are admitted. Also, each car has a driver!)

6 Flippin' Coins

Suppose we have an unbiased coin, with outcomes H and T , with probability of heads $\mathbb{P}[H] = 1/2$ and probability of tails also $\mathbb{P}[T] = 1/2$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3) , where $X_i \in \{H, T\}$.

- (a) What is the *sample space* for our experiment?
- (b) Which of the following are examples of *events*? Select all that apply.

- $\{(H, H, T), (H, H), (T)\}$