3. (**30 points**) Following is an initial tableau of a linear programming problem (LP) in standard form.

x_1	x_2	x_3	x_4	x_5	x_6	RHS
2	b	3	-1	1	0	8
a	c	-2	e	0	1	20
3	d	-11	7	0	0	0

Using simplex method to solve it, after some iterations, we get the following tableau (T^*).

(T^*)

x_1	x_2	x_3	x_4	x_5	x_6	RHS
$\frac{11}{5}$	$\frac{13}{10}$	0	1	$\frac{1}{5}$	i	k
f	$\frac{21}{10}$	1	0	g	$\frac{1}{10}$	l
3	12	0	0	h	j	m

(a) Find the values of $a \sim m$.

[Use the values you obtained in (a) for the following questions.]

(b) What is the current solution $(x_1, x_2, x_3, x_4, x_5, x_6)^T$? What is the current objective function value?

(c) Is (T^*) an optimal tableau? If not, find an optimal solution starting from (T^*).

(d) Find the dual of this LP and its optimal solution (the objective value and the value of the dual variables including structural, surplus/slack variables, if any). [Use the tableau - do not solve from scratch!]

[Problem (e)-(h) should be done independently.]

(e) Suppose $\mathbf{b}$, the RHS coefficient vector of these two constraints, is changed to $(-9, 15)^T$. What is the effect of this change on the solution?

(f) Would the optimal solution be altered, if $\mathbf{a}_2$, the second column of the coefficient matrix, is changed to $(4, 2)^T$? If yes, find the new optimal solution.

(g) What effect will the addition of the following constraint have on the solution?

$$x_1 + x_2 \geq 2$$

(h) Find the largest and smallest value of λ such that the given solution is still optimal if the cost vector $\mathbf{c}$ in the objective function is replaced by $\mathbf{c} + \lambda \mathbf{1}$, where $\mathbf{1} = (1, 1, 1, 1, 1, 1)^T$.