6. (**10 points**) Considering the following linear programming problem (LP)

$$
\begin{aligned}
\text{Maximize} \quad & z = \sum_{j=1}^{n} c_j x_j \\
\text{Subject to} \quad & \sum_{j=1}^{n} a_{ij} x_j = b_i, \quad i = 1, 2, \ldots, m && (1) \\
& x_j \leq u_j, \quad j = 1, 2, \ldots, n && (2) \\
& x_j \geq 0, \quad j = 1, 2, \ldots, n
\end{aligned}
$$

where $u_j > 0$, $j = 1, 2, \ldots, n$. Let y_i and z_j be dual variables associated with constraints (1) and (2) respectively.

Let

$$R_j = c_j - \sum_{i=1}^{m} a_{ij} y_i.$$

Show that $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$ is an optimal solution to this LP if and only if

$$
\begin{cases}
R_j \leq 0, & \text{if } x_j = 0, \\
R_j = 0, & \text{if } 0 < x_j < u_j, \\
R_j \geq 0, & \text{if } x_j = u_j.
\end{cases}
$$

—————— The End ——————