

ECMT3170: Computational Econometrics

Assignment 2

Due by 9AM on Monday, 4 May 2020

Practicalities. Feel free to use any software you are comfortable with, both for the computations and to type up your answers. Partly handwritten answers are fine too, as long as they are reasonably legible; just scan or take a picture of those parts. Your submission needs to contain both your computer code and your answers to all parts of both questions; there are five such parts, worth three marks each. Please submit your solutions using the submission tool on Canvas. In order to comply with our anonymous marking policy, make sure that your submission contains your SID but not your name.

Question 1. Reconsider the following problem, which we also studied in the first assignment. We are given a sample x_1, x_2, \dots, x_n where the observations are independently generated from a distribution with support $[a, b]$. We wish to test $H_0 : a = 0 \text{ and } b = 1$, against the general alternative $H_a : \text{not } H_0$. You may assume that all observations satisfy $0 \leq x_i \leq 1$, because otherwise formal testing is pointless and we might as well reject the null hypothesis immediately. The test statistic is still $W = -2n \ln \left[\frac{\max\{x_i\} - \min\{x_i\}}{b - a} \right]$ and rejection happens if W is larger than some critical value, which is to be determined.

We continue to work with samples of size $n = 10$, and we are interested in the size of a test with nominal level 5%. We also still want to know the power of this test against the three different alternatives where $[a, b]$ is either $[0, 0.9]$, $[0.1, 1]$, or $[0.1, 0.9]$.

In the first assignment, we made the additional assumption that the distribution of the x_i was uniform. We do not wish to make that assumption any more. This means that our techniques from the first assignment are no longer valid, and we need to use either the nonparametric bootstrap or subsampling to perform our test.

Despite the fact that we do not wish to make any assumptions on the distribution of x_i when performing a test, we will need to sample from some DGP in order to perform a simulation study. You may use the following DGP: let u_i be standard uniform as usual, and set $x_i = a + (b - a) u_i^4 (5 - 4u_i)$. The distribution of this x_i has the required support $[a, b]$ but a shape that is far from uniform.

- (a) Implement a nonparametric bootstrap test for this problem, and perform the size and power study.
- (b) Implement a subsampling-based test for this problem, and perform the size and power study.

Question 2. In this problem, we attempt to estimate the parameters of the following distribution, which may be used to model individuals' net wealth (i.e. assets minus debt). People make different choices in life and experience different random shocks, so we may model their net wealth as a sample of independent draws from a normal distribution.

However, there is some obvious heterogeneity. For some people this normal distribution has a higher mean, simply because they were born into a richer country or a richer family. Likewise, for some people this normal distribution has a higher standard deviation, because they take more risks. All things considered, we end up with $y_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$, where μ_i and σ_i are drawn (independently from each other, and independently across individuals) from the distributions with the following PDFs:

$$p(\mu_i) = \frac{1}{\alpha\beta} \exp\left[-\frac{\mu_i}{\alpha} - \frac{1}{\beta} \exp\left[-\frac{\mu_i}{\alpha}\right]\right] \quad \text{and} \quad p(\sigma_i) = \begin{cases} \frac{1}{\gamma\delta} \sigma_i^{\frac{1}{\delta}-1} \exp\left[-\frac{1}{\gamma} \sigma_i^{\frac{1}{\delta}}\right] & \text{for } \sigma_i > 0; \\ 0 & \text{elsewhere.} \end{cases}$$

We are interested in estimating the four parameters α , β , γ , and δ . All of them are positive; there are no other restrictions.

(a) As you can probably see coming, we will need to be able to sample from the distributions of μ_i and σ_i . It turns out that the inverse CDF method works in both cases – for σ_i you should find this fairly easy, and for μ_i you may want to look back at tutorial 6, where a very similar distribution was used. Write two programmes: one that takes standard uniform draws $u_i^{(s)}$, α , and β as input, and produces draws $\mu_i^{(s)}$ as output; and one that takes standard uniform draws $v_i^{(s)}$, γ , and δ as input, and produces draws $\sigma_i^{(s)}$ as output. The following is not part of the assignment, but you may check that your code works by verifying that it produces histograms that are similar to the plots at the bottom of this page.

For the remaining two parts of this assignment, use the data set `wealth` that is provided with the assignment. It contains the net wealth reported in a survey of 418 people, in whichever currency these numbers make sense, probably thousands of 1980 dollars. Estimate all four parameters of the model, and provide a standard error for each estimate. Please do not bother with analytical derivatives, as those look like very boring busywork.

(b) Use simulated maximum likelihood. That is, decide what the latent variable η_i should be, write down a formula for your subsimulator, and implement the procedure in your programming language of choice.

(c) Use the method of simulated moments. That is, decide on four useful moment functions (no need to do more than four) and implement the procedure in your programming language of choice.

