Q 1 (a) If X and Y are two independent normal random variables with mean 0 and deviation $\sqrt{2}$. Prove (by using convolution of density functions) that $Z = X + Y$ is also a normal random variable. Also, find the mean and standard deviation of Z.

(b) In a group of 10 people, there are 4 girls and 6 boys. If a sample of 3 people is chosen at random. Find the expected number of boys chosen?

Q 2 The time (in hours) to repair a machine A is exponentially distributed random variable X with mean 1 and the time (in hours) to repair a machine B is exponentially distributed random variable Y with mean 1/2.

Find (a) Probability density function of $Z = X\text{-}Y$

(b) Find the probability that repair time for machine A is at most 3 hours more than the repair time for machine B.

Q3 (a) A manufacturer produces a non-defective item with probability 0.96. Approximate the probability that of the next 100 items produced, less than 6 would be defective.

(b) Suppose as per some data, 32% of the young adults believe in astrology and 35% of older adults do not believe in astrology. Suppose a sample of 50 young and 50 older adults is chosen. Approximate the probability that number of people who do not believe in astrology is at least as large as number of people who believes in astrology.

Q4 (a) Let X and Y be two independent random variables distributed uniformly on the interval [0, 4]. What is the conditional probability of $Y \geq \frac{1}{2}$, given $Y \geq 1-2X$.

(b) Suppose that the expected number of calls received at a call center is 8 per hour. What is the probability that in next 20 minutes, you would receive 3 or more calls?

Q5 (a) Suppose that a random variable X, giving the lifetime of an electronic device, has a cumulative distribution function given as

$$F(x) = \begin{cases} \ln x & x > 0 \\ 0 & x \leq 0 \end{cases} \qquad \text{Find } E\left[\sqrt{x}\left(e^{-\frac{x}{2}}\right)\right]$$

(b) Alex and John are playing a game which is best of 7 series, which means the two guys play each other until one of them wins 4 games. The probability that the John win a given game is 0.45. Assuming outcomes are independent, what is the probability that John wins the series?

Q6 (a) We flip a fair coin three times. Let X be the number of heads observed, Y the number of heads in the last 2 flips.

(i) Find the joint probability mass function of (X, Y).

(ii) Write down the marginal probability mass function for Y.

(iii) Are X and Y independent? Give reasons.

(iv) Compute $P(X+Y>2)$

(v) Compute $P(X>1|Y=1)$

(b) If X and Y are two independent variables and Var (X) = 2 and Var(Y) = 3, Find Var (5X- 4Y + 2)