College of Sciences | **كلية العلوم**

Department of Statistics and Operation Research | **قسم الإحصاء وبحوث العمليات**

Home work: From satudrday 28/3 o Monday 30/03 before 12 AM

Student's Name		اسم الطالب
ID number		الرقم الجامعي
Section No.		رقم الشعبة
Classroom No.		رقم قاعة الاختبار
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Roll Number		رقم التحضير

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Question 1:

Let $X_1, X_2, \dots, X_n$ be a random sample has from a continuous distribution with probability density function given by

$$f(x,\mu) = \frac{1}{2\sqrt{\pi}} e^{-\frac{1}{4}(x-\mu)^2}$$

Find:

1. The method of moments estimator of μ.
2. T: The maximum likelihood estimator of μ.
3. Prove that T is consistent.
4. The ML estimate of μ, when $x_1 = 3.4, x_2 = 2.5, x_3 = 3.1, x_4 = 3.2, x_5 = 2.2,$
   $x_6 = 2, x_7 = 2.6 \ , x_8 = 2.$

Question 2:

Suppose $X_1, X_2, \cdots, X_n$ are i.i.d. random variables with density function $f(x|\sigma) = \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right)$, please find the maximum likelihood estimate of σ.

Question 3:

We consider a sample $X_1, X_2, .., X_N$ of i.i.d. discrete random variables, where X_i has a geometric distribution with a pmf given by:

$$f_X(x,\theta) = \Pr(X = x) = \theta \times (1-\theta)^{x-1} \qquad \forall x \in \{1,2,3,..\}$$

where the success probability θ satisfies $0 < \theta < 1$ and is unknown. We assume that:

$$\mathbb{E}(X) = \frac{1}{\theta} \qquad \mathbb{V}(X) = \frac{1-\theta}{\theta^2}$$

a) *Write the log-likelihood function of the sample $\{x_1, x_2, ..x_N\}$*

b) *Determine the maximum likelihood estimator of the success probability θ.*

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If $X_1, X_2, \dots, X_n$ be independent identical random variables with pdf

$$f(x, \theta)=(2x)/\theta^2, \quad 0 < x < \theta.$$

1) Find:

1. The method of moments estimator of θ.

2. The maximum likelihood estimator of θ.

3. The mean of $T = \bar{X}$.

4. The MSE of T.

2) Let S=3T/2. Prove that

   1. S is an unbiased estimator of θ ;
   2. S is consistent estimator of θ.

Answer:

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